Instructional Strategies Used By Effective Mathematics Teachers In Adventist Elementary Schools In Florida: An MQI Analysis

Anna Adkins

This research is a product of the graduate program in Leadership PhD at Andrews University. Find out more about the program.
ABSTRACT

INSTRUCTIONAL STRATEGIES USED BY EFFECTIVE MATHEMATICS TEACHERS IN ADVENTIST ELEMENTARY SCHOOLS IN FLORIDA: AN MQI ANALYSIS

by

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Chair: Shirley Freed
ABSTRACT OF GRADUATE STUDENT RESEARCH

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Andrews University
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Title: INSTRUCTIONAL STRATEGIES USED BY EFFECTIVE MATHEMATICS TEACHERS IN ADVENTIST ELEMENTARY SCHOOLS IN FLORIDA: AN MQI ANALYSIS

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The Problem

Teaching mathematics can be a difficult task. United States students on comparative international mathematics assessments consistently rank near the test averages. On a national assessment, commonly called the Nation’s Report Card, more than half of American students rank as ‘not proficient’ in mathematics. The purpose of this study was to describe the strategies used by teachers with at least average mathematical knowledge for teaching. This was evidenced by their scores on the Mathematical Knowledge Test for Teaching (MKT). In addition to teacher knowledge, the study sample was selected from among schools where the five-year average was
above the average curve equivalent mean. The research question was “In what ways do teachers of high achieving mathematics students deliver mathematics instruction?”

Method

The methodology employed was a case study. The research analyzed videos of eight mathematics teachers using the Mathematics Quality of Instruction (MQI), to analyze the quality and the usage of the instructional strategies. The MQI provided a framework to view and analyze instruction in 14 mathematics lessons in grades 1 – 8. The MQI is divided into four dimensions: Richness of the Mathematics, Working with Students and Mathematics, Teacher Error and Imprecisions, and Common Core-Aligned Student Practices. Each dimension is further divided into instructional strategies. The Richness of the Mathematics dimension has six instructional strategies: Linking Between Representations, Teacher Provided Explanations, Mathematical Sense-Making, Multiple Procedures or Solutions, Patterns and Generalizations, and Mathematical Language. Working with Students and Mathematics has two strategies: Remediation of Student Errors and Difficulties and Teacher Uses Student Mathematical Contributions. Teacher Errors and Imprecisions has three parts: Mathematical Imprecision in Language or Notation, Content Errors, and Lack of Clarity in Presentations of Mathematical Content. Common Core-Aligned Student Practices has five strategies: Students Provide Explanations, Student Mathematical Questioning and Reasoning, Students Communicate about the Mathematics of the Segment, Task Cognitive Demand, and Students Work with Contextualized Problems. The lessons were videotaped and then 7.5-minute segments were coded using the MQI framework.
Results

These teachers know their mathematics. Of the 80 segments analyzed only 3% contained any errors and all instruction was clear. Of the four MQI dimensions, the teachers excelled in the Richness of the Mathematics dimension. Most of the strategies in this section were used a large majority of the time. When utilizing Working with Students and the Mathematics, the teachers used student comments and provided correction to student errors when needed. Common Core-Aligned Student Practices was the least used dimension. While students did communicate mathematically, the level of their communication tended to be procedural. There were a few instances of students making conjectures or conclusions based on patterns or mathematical reasoning.

The five strategies that were used more than 80% of the time were Mathematical Language (90%), Teacher Uses Student Mathematical Contributions (90%), Students Communicate about the Mathematics (85%), Mathematical Sense-Making (83%), and Teacher Provided Explanations (80%).

The least used strategies were Multiple Procedures or Solutions (45%) and Student Mathematical Questioning (45%). Remediation of Student Errors was used in 51% of the segments, likely because students made few errors. Linking Between Representations, Patterns and Generalizations, Students Provide Explanations, and Students Work with Contextualized Problems were found in 61% of the segments.

There was higher quality of usage in grades one and two for the Richness of the Mathematics dimension. Teachers in grades three-five used both Working with Students and Mathematics strategies more often than the other grades. The quality of strategy
usage by grade showed that in all strategies, grades one and two had a higher quality of usage than the other grades.

Conclusions

This study was conducted to identify the strategies used by effective math teachers in the Florida Conference of Seventh-day Adventists. These teachers have evident knowledge and are skillful in explaining and providing examples of the mathematical content. Their instruction was clear and precise. A relative weakness of the study teachers was their ability to guide students to generalize and reason mathematically. The ability to express oneself mathematically will increase student understanding and develop students who are successful in elementary school mathematics and beyond.
Andrews University

School of Education

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A Dissertation
Presented in Partial Fulfillment
of the Requirements for the Degree
Doctor of Philosophy

by

Anna Adkins

October 2017
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Date approved
DEDICATION

This work is dedicated to Cherise Anne Adkins (September 9, 1980 – September 13, 2000).

“She walks in beauty, like the night
Of cloudless climes and starry skies;
And all that’s best of dark and bright
Meet in her aspect and her eyes;
Thus mellowed to that tender light
Which heaven to gaudy day denies…

And on that cheek, and o’er that brow,
So soft, so calm, yet eloquent,
The smiles that win, the tints that glow,
But tell of days in goodness spent,
A mind at peace with all below,
A heart whose love is innocent!”

(Lord Byron, 2012, p. 612)
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<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Full Form</th>
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<tbody>
<tr>
<td>CAT</td>
<td>Computerized Adaptive Testing</td>
</tr>
<tr>
<td>CCSS</td>
<td>Common Core State Standards</td>
</tr>
<tr>
<td>ITBS</td>
<td>Iowa Test of Basic Skills</td>
</tr>
<tr>
<td>LMT</td>
<td>Learning Mathematics for Teaching</td>
</tr>
<tr>
<td>MET</td>
<td>Measure of Effective Teaching</td>
</tr>
<tr>
<td>MKT</td>
<td>Mathematical Knowledge for Teaching</td>
</tr>
<tr>
<td>MQI</td>
<td>Mathematical Quality of Instruction</td>
</tr>
<tr>
<td>NAEP</td>
<td>National Assessment of Educational Program</td>
</tr>
<tr>
<td>NBPTS</td>
<td>National Board for Professional Teaching Standards</td>
</tr>
<tr>
<td>NCE</td>
<td>Normal Curve Equivalent</td>
</tr>
<tr>
<td>NCTE</td>
<td>National Center for Teacher Effectiveness</td>
</tr>
<tr>
<td>NCTM</td>
<td>National Council of Teachers of Mathematics</td>
</tr>
<tr>
<td>OECD</td>
<td>Organization for Economic Co-operation and Development</td>
</tr>
<tr>
<td>PCK</td>
<td>Pedagogical Content Knowledge</td>
</tr>
<tr>
<td>PISA</td>
<td>Program for International Student Assessment</td>
</tr>
<tr>
<td>SES</td>
<td>Socioeconomic Status</td>
</tr>
<tr>
<td>TIMSS</td>
<td>Trends in International Math and Science Study</td>
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<tr>
<td>TKAS</td>
<td>Teacher Knowledge Assessment System</td>
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</table>
ACKNOWLEDGMENTS

There are no words that can truly express how I feel. Thank you is not big enough. Appreciate seems to lack gravitas. Grateful seems hollow. So, how do I write this? I’m stuck with the first line of an Andre Crouch song, “My Tribute”, “How do I say thanks?” (Crouch, 1972).

According to the Online Etymology Dictionary (Harper, 2017) thank comes from a Middle English word related to think, (like song and sing). Thus, etymologically it means having good or pleasant memories. The same source defines appreciate, as a word derived from the Latin word *appretiare*, which means to determine the value to an item. Later the word came to imply placing a positive value on something. Gratefulness comes from the Latin *gratus* meaning something pleases according to the Online Etymology Dictionary. In the 1550’s the word was used to describe things that pleased the mind and also as having the disposition to repay someone with actions for their help. Now, with a better understanding of the vocabulary, I feel confident in writing this acknowledgement.

To each person who has helped me in this process, I would like to say I have pleasant memories of your support and encouragement. I place great value on what you have done for me. And, I am ready and willing to repay what you have given me with my heart and soul.

Christopher and Lui Na I thank you, appreciate, and am grateful for your love. It is the fire that keeps me warm. Your support and belief in me is uplifting. I thankfully appreciate all you taught me Cherise, because you were a curious learner I learned to see

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the world through your eyes. I gratefully appreciate and thank Jack because you encouraged me to write and gave me an audience to practice my writing. Dr. Ledesma your confidence in my ability has always been greater than my own. You always saw my possibilities when I only saw my limitations. Dr. Freed what a wonderful teacher you are. You can push for excellence while creating a safe environment for each student. You have been a patient and gentle guide though out this process. You are my “Virgil” through in this dissertation process.

Now like an Oscar recipient who has over stayed their time on stage and is being ushered off the stage, I’m now subjected to naming the others. I want to give my eternally grateful and appreciative thanks to the Florida Conference of Seventh-day Adventist; Doctors Henry-Saturne and Lundgren for your time and support in the improvement of my research; Dennis and Nicole, editors; my Mother; Rose, Iris, Tony-my siblings and their families, Joan, Valmae, Rick, Christian, and Ariana.
CHAPTER 1

INTRODUCTION

Background of the Problem

An armistice has been reached in the reading wars. The parties, (Smydo, 2007; Veenman, Hout-Wolters, & Afflerbach, 2006) whole language versus phonemic methods for learning to read), have joined forces and come to the conclusion that both whole language and phonemic instruction are necessary for students to become competent readers (Langenberg et al., 2000). According to the National Mathematics Advisory Panel (Benbow, 2008), the American educational system is now fighting on a new front. The report emphasizes the need for more research in mathematics education. They list specific areas for study such as teacher knowledge, effective instructional strategies, student effort and motivation, and mathematics learning process.

The struggle to make American students successful in mathematics has been going on for half a century (Klein, 2007). The opening salvo of the “Math War” was the launch of the Sputnik satellite (Burris, 2005; Klein, 2007). Before the launching of Sputnik, education had two major branches: classical education and progressive education. The early years of the 20th century brought the contrasting ideas of John Dewey and E. L. Thorndike. Dewey’s vision of mathematics instruction was that it (mathematics) should be taught by “connect[ing mathematical] experience [to] real-life” (Davison, Mitchell, & Montana, 2008, p. 146). Conversely, Thorndike emphasized
“recitation and rote memorization followed by measurement of outcomes through achievement testing” (Davison et al., 2008, p. 146). The launching of Sputnik brought these competing philosophies regarding mathematics instruction to the national consciousness.

During this time, the National Science Foundation developed a new mathematics curriculum, which became known as the “New Math.” The new curricula had “aspects of set theory, modular arithmetic and symbolic logic embedded in the curriculum” (Schoenfeld, 2004). However, the “New Math,” created by academics, did not receive buy-in from the primary stakeholders: teachers and parents. The American experience with “New Math” led to the back-to-the-basics movement of the 1970’s. However, after ten years of instruction in the basics, American students “showed little ability at problem-solving . . . but performance on the ‘basics’ had not improved either” (Schoenfeld, 2004, p. 258).

The 1980s and 90s brought rhetorical battles over mathematics instruction with each side claiming victory while very little research was done to support their assertions. In 1989 the National Council of Teachers of Mathematics (NCTM) developed “The Standards” (Klein, 2007). These standards were to provide educators with an “agenda for actions” (Klein, 2007, p. 22). “The new standards focused greater attention to operation sense, the use of calculators for complex computation, collection and organization of data, pattern recognition and description, the use of manipulative materials and cooperative work, and decreased attention to long division, paper and pencil fraction computation, rote practice, and teaching by telling” (Klein, 2007, p. 23). In June of 2010, the National Governors Association and the Council of Chief State School Officers
released the Common Core State Standards (CCSS) in mathematics (Mathis, 2010). The CCSS allowed states to collaborate in the development of the standards. The goal of the CCSS was to provide the “standards necessary for national economic competitiveness in a global economy” (Mathis, 2010, p. 2). The CCSS also gave states a way to “collectively create and share high-quality tools such as assessments, curricula, instructional materials, and professional development programs” (Jamison, 2000, p. 165).

Although the view that mathematics is a dynamic and exploratory discipline is currently agreed upon (Henningsen & Stein, 1997), reviewers of the CCSS such as Mathis (2010) and Zhao (2009) still contend that the new standards are not sufficient in order to achieve the goal of improving mathematics knowledge in the United States. According to Mathis (2010), the NCTM has concerns that these new standards do not provide enough emphasis in certain areas (such as critical thinking skills), and places too much on others (such as fractions). In his review of the CCSS Mathis, (2010) claims there is not enough evidence that national standards will improve student achievement or international economic competitiveness. The debate remains the same with some researchers calling for a back-to-basics curriculum (Klein, 2007; Loveless, 2001) and other calls for a curriculum which calls for standards-based teaching (Draper, 2002; Fuson, Carroll, & Drueck, 2000; Huntley, Rasmussen, Villarubi, Sangtong, & Fey, 2000). While there has been constant discussion on the subject, student scores have not shown sufficient gains (Hechinger, 2008; Ornstein, 2010).

The Problem Statement

Low mathematics scores are a national problem. In general, Seventh-day Adventist schools’ mathematics scores on the Iowa Test of Basic Skills (ITBS) are lower
than any other area. While many schools have low mathematics achievement, this is not the case in all Seventh-day Adventist schools (Kido, Thayer, & Cruise, 2008). According to “CognitiveGenessis” (Kido, 2017), a study of 51,706 students between 2006 to 2009, Adventist students scored at the 70th percentile in Language Arts and in the 62nd percentile in mathematics. Seventh-day Adventist teachers consistently have students who score in the top quartile in mathematics. However, it is unknown how these teachers deliver their mathematics instruction. A deeper understanding of the characteristics of these teachers may shed light on the problem of low mathematics scores.

**Purpose of the Study**

The purpose of this study was to identify schools where the average student score in mathematics is at the 50th Normal Curve Equivalent (NCE) score or above within the Florida Conference of Seventh-day Adventists and observe the mathematics teachers to develop an understanding of how these teachers approach mathematics instruction.

**Research Question**

The question guiding this study was: In what ways do elementary teachers of high achieving mathematics students deliver mathematics instruction?

**Research Design**

Using a case study design, this study describes the teaching patterns of teachers in schools where the students are achieving at or above the 50th percentile. This study describes what a sample of highly effective teachers do in their classrooms. Each teacher in this study took the “Mathematical Knowledge for Teaching Test” (Schilling & Hill, 2007) developed by the Learning Mathematics for Teaching (LMT) Project at Michigan
State University. The Mathematical Knowledge for Teaching (MKT) measures a teacher’s MKT. The teachers in this study scored above the mean on one or both sections of the MKT.

A purposive sample of the pool of Florida Conference teachers was obtained. The sample consisted of teachers whose schools met the ITBS criteria and whose scores on the MKT met the criteria. These teachers were videotaped. Then teacher videos were observed using the Mathematical Quality of Instruction (MQI) rubric to describe teaching practices and strategies during mathematics instruction. The MQI rubric was developed by the LMT/Mathematics Instrument Development Group at Harvard University Graduate School of Education. Its purpose is to identify quality instruction in mathematics.

**Study Significance**

The results of this study may be used to inform educational leaders about best practices in mathematics instruction. This study creates a clearer picture of the instructional characteristics of successful teachers, which may lead to the development of professional training and teacher education programs. Eventually, this knowledge could inform teachers about their practices, so they can help their students learn and enjoy mathematics.

**Conceptual Framework**

The theoretical framework of this study rests in the work of teaching. The work of a teacher according to Ball and Forzani (2009) is “the core tasks that teachers must execute to help pupils learn” (Ball & Forzani, 2009). The work of teaching is everything a teacher does so students will learn. The work of teaching is comparable to the work of
other clinical practice professionals such as doctors (Alter & Coggshall, 2009; Cohen, 2005). Like other clinical professions, the work of teaching includes students and their individual needs. “Teaching practice is constructed in the interplay of mathematics, students, and pedagogy” (Ball & Bass, 2000, p. 104).

A teacher’s success is dependent on the engagement of students. In a talk given at Vanderbilt University, Deborah Ball (Vanderbilt University, 2008), describes the work of teaching as complex and requiring special skills, as well as specific theoretical and technical knowledge not possessed by laypeople. Like other clinical practice professionals, teachers use knowledge of clients to make informed decisions about the next course of action (Ball & Forzani, 2009). Teachers do not work alone but within professional organizations. Finally, like other clinical practice professionals, teachers are granted the right to teach after training and must continually participate in ongoing learning. Ball (Vanderbilt University, 2008) also describes teaching as an unnatural act. Classroom teaching is not just helping others do things; teaching requires one to do things not done in everyday life. Teachers ask questions they may not know the answer to. Teachers may ask others to do things they don’t know how to do. Teachers must listen and probe the thinking of others. The work of teaching is also intricate since it involves knowledge of the material, the student, the environment, and the curriculum (Ball & Forzani, 2009; Schoenfeld, 1998).

In normal conversations, people do not explain ordinary information that listeners understand (Brophy, 1986). According to Darling-Hammond (2000), explanations are not given in routine conversations unless the validity of the statement is called into question, or if the listener needs more or new information on the subject. Henningsen and Stein
(1997) describes classroom explanations where students make conjectures, support
claims with evidence, and use representations to describe mathematical concepts. During
classroom discussions, teachers re-voice student claims. Teachers ask students to re-
explain what they have heard. Teachers encourage students to develop multiple solutions
for simple problems. These are just a few examples which demonstrate teaching is not a
natural skill. Deborah Ball calls the ability to have both expertise in a subject and
understand how to break it down into its component parts “the unnatural act of teaching
where teachers use professional knowledge and skill to make these interactions [between
student and content] most productive of students’ learning” (Ball & Forzani, 2009, p. 449).

Hanushek and Rivkin (2006) and Pang (2009) have described the characteristics
of effective teaching as the relationship between the teacher’s actions and the students’
ability to construct mathematical models. Dehaene and Cohen (1997) call this framework
the cognitive research approach, which looks for the planning, interactive decision-
making, and judgments that teachers make. My research describes how teachers conduct
their instructional practices, “the work of teaching” by focusing on teacher knowledge
and teacher choices, as the processes that guide students to success in mathematics.

**Assumptions**

I made several assumptions in planning my research. First, I assumed that the
teachers of high achieving mathematics students would be using strategies that other
teachers would want to emulate. I assumed that using the MQI, I would be able to
describe these teachers’ behaviors in enough detail so that others could learn their
strategies. I also assumed that mathematics teachers need adequate knowledge about
math before they can teach mathematics well. I also assumed that the scheduled videotaping of the classroom teaching would represent how the teachers typically taught their students to achieve above average on national tests.

**Delimitation**

The scope of this research was within the Florida Conference of Seventh-day Adventists where average school scores on the total mathematics portion of the ITBS were at or above the 50th percentile of the Conference. This research does not describe teachers whose students were below average or teachers outside the Florida Conference or teachers who did not pass at least one of the two parts of the MKT test.

**Limitations**

This study was limited to the teachers in the qualifying pool who were available. As a researcher, I am aware that the task of teaching is complicated and all observation protocols are subject to assessing some irrelevant construct to teacher quality. Irrelevant construct variables include all or some of the following: student population, teacher beliefs, adopted curriculum, school and district leadership, and lessons observed. Another limitation of this study is that teachers knew ahead of time when I would be videotaping their class, and this could have influenced their choice of lessons and modified their teaching.

**Definitions of Terms**

*Absolutism*: the philosophical view that mathematical knowledge “is an objective, absolute, certain and incorrigible body of knowledge which rests on the firm foundations of deductive logic” (Ernest, 1991, p. 5).
**Common Content Knowledge:** the mathematical “knowledge of a kind used in a wide variety of settings – in other words not unique to teaching”; these are not specialized understandings but are questions that typically would be answerable by others who know mathematics” (Ball, Thames, & Phelps, 2008).

**Common Core State Standards for Mathematics:** “The knowledge and skills students need to be prepared for mathematics in college, career, and life are woven throughout the mathematics standards” (National Governors Association Center for Best Practices Council of Chief State School Officers, 2010a).

**Common Core-Aligned Mathematical Practice:** “develop student adaptive reasoning, strategic competence, conceptual understanding, procedural fluency, and productive disposition” (National Governors Association Center for Best Practices Council of Chief State School Officers, 2010a).

**Constructivism:** “the individual knower through mental activity constructs a theory of learning that states that knowledge” (Ernest, 1991, p. 11).

**Effective Teachers:** an effective teacher of mathematics develops student ability to apply mathematical content knowledge. According to NCTM (2014), applying mathematical content includes solving problems, reasoning mathematically and providing proofs, communicating with others, making connections between mathematical standards, and understanding and using a variety of mathematical representations. The Common Core (National Governors Association Center for Best Practices Council of Chief State School Officers, 2010b) Standards of Mathematical Practices also provide a description of what students can do including making sense of problems and persevering in solving them, reasoning abstractly and quantitatively, constructing viable arguments
and critiquing the reasoning of others, modeling with mathematics, using appropriate
tools strategically, attending to precision, and looking for and making use of
mathematical structures.

Fallibilism: “views mathematics as the outcome of social processes. Mathematical
knowledge is understood to be eternally open to revision, both in terms of its proofs and
its concepts” (Ernest, 1991, p. 18).

Fluent Performance: “based on understanding of the routine that is being carried
out” (Marshall, 2003, p. 200).

Mathematical Knowledge for Teaching: “the mathematical knowledge needed to
carry out the work of teaching mathematics” (Ball, Thames, et al., 2008).

Mechanical Performance: “performance by rote in which the necessary
understanding is not present” (Marshall, 2003, p. 200).

Mathematical Understanding: “the ability to justify, in a way appropriate to the
student’s mathematical maturity, why a particular mathematical statement is true or
where a mathematical rule comes from” (National Governors Association Center for Best

Mathematical Thinking: the ability to solve mathematical problems; communicate
ideas concretely, pictorially, or abstractly; and understand the mathematical ideas of
others.

Pedagogical Content Knowledge (PCK): the ability of a teacher to take in-depth
knowledge of a content and make it accessible to students.(Turnuklu & Yesildere, 2007,
p. 2).
Organization of the Study

This study is organized into six chapters. The first chapter is an introduction to the study. The second chapter reviews the relevant literature related to the study. The third chapter describes the methods used to conduct the research. The fourth chapter is a rich description of mathematics lessons enacted by the teachers and students. The fifth chapter provides a detailed comparison of the usage and quality of teaching strategies. In the final chapter I analyze the results, summarize the findings and implications, and provide recommendations for practice and future research.
CHAPTER 2

LITERATURE REVIEW

Introduction

To understand excellence in mathematics education, this literature review addresses six major topics. I review the literature on student achievement, wherein I describe American student mathematics achievement on international and national tests. That is followed by a review of theories describing human comprehension of mathematics from neurological and cognitive perspectives. Next, I review research on effective mathematics instructional strategies. Armed with an understanding of what it means to know mathematics and best instructional methods, in the final section I look at the characteristics of effective teachers and how these teachers have been identified in past research.

Student Achievement and Student Learning

It is important to understand the general achievement of American students when compared to other countries using the Program for International Student Assessment (PISA) and Trends in International Math and Science Study (TIMSS) studies. Also, comparisons of American students by states using the National Assessment of Educational Program (NAEP) provide the context for this study and help to highlight the rationale for this study.
Current International Mathematics Assessment Comparisons

Since 1995, the National Center for Education Statistics (NCES) has collected data for the TIMSS. Trends in International Math and Science Study (Gonzales, Jocelyn, Roey, Kastberg, & Brenwald, 2008) is a cross-national comparative study of student achievement in mathematics and science of fourth- and eighth-graders. Tests are given every three years. In 2007, the fourth-grade test was given in 43 jurisdictions, and the eighth-grade test was given in 56 jurisdictions.

The International Association for the Evaluation of Education Achievement provides each jurisdiction with detailed guidelines to ensure comparability of data. Any school within a country containing fourth or eighth-grade can be considered as part of the target population. The 2007 study used a three-stage stratified cluster sample to ensure validity. Trends in International Math and Science Study test items were developed cooperatively by members of the participating countries with input from experts in mathematics and science. Mathematics items consisted of questions in numbers, algebra, geometry, data, and probability. Students and teachers also completed background questionnaires. In addition to scaled scores, the international committee provided descriptive benchmarks (See Table 1).

Trends in International Math and Science Study authors reported the scaled scores for the participating countries. The average United States mathematics scaled score was 528 for fourth-grade and 508 for eighth-grade. While students in both grades scored above the test average, they were not statistically above the average. In the fourth-grade test, the countries of Hong Kong, Singapore, Chinese Taipei, Japan, Kazakhstan, Russian Federation, England, and Latvia were above the average. Eighth-grade students from
Table 1

Percentage of Students Scoring at Each Level of the PISA

<table>
<thead>
<tr>
<th>Score</th>
<th>Benchmark</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>625</td>
<td>Advanced International</td>
<td>Students can apply their understanding and knowledge in a variety of relatively complex situations and explain their reasoning.</td>
</tr>
<tr>
<td>550</td>
<td>High International</td>
<td>Students can apply their knowledge and understanding to solve problems.</td>
</tr>
<tr>
<td>475</td>
<td>Intermediate International</td>
<td>Students can apply basic mathematical knowledge in straightforward situations.</td>
</tr>
<tr>
<td>400</td>
<td>Low International</td>
<td>Students have some basic mathematical knowledge.</td>
</tr>
</tbody>
</table>

*Note: From (Mullis, Martin, & Foy, 2008, p. 68)*

China, Korea, Singapore, Hong Kong and Japan scored higher than United States (Gonzales et al., 2008).

Huang (2009) looked at the TIMSS score from a different angle. He looked at a country’s measures of variability, skewness, performance variability, and distribution of performance for the 1999 TIMSS. The distribution of performance is the largest in the United States. This indicates that American classrooms are quite dissimilar from one another. When looking at variation within classrooms the United States is in the middle with high-performing countries like Singapore, the Netherlands, and the Flemish Region of Belgium – all with the least variation within classrooms performance. “Higher performing countries in mathematics, such as Singapore, South Korea, Hong Kong, Japan, the Flemish Regions of Belgium and the Netherlands are generally characterized by [a] more equal performance among students within countries” (Huang, 2009, p. 339).
Using a different measure, the PISA coordinated by the Organization for Economic Co-operation and Development (OECD) has developed a quantitative study that investigates 15-year-olds worldwide every three years. Every period of assessment tests students’ performance in mathematics, science, and reading with a focus on one of the three competency fields of reading, mathematics, or science in each testing cycle. In 2003, mathematics was the focus, with 41 countries participating. “PISA uses a concept of mathematical literacy that is concerned with the capacity of students to analyze, reason, and communicate effectively as they pose, solve, and interpret mathematical problems in a variety of situations involving quantitative, spatial, probabilistic, or other mathematical concepts” (Schleicher, 2007, p. 2). Program for International Student Assessment reports student performance in six levels with level 2 as the baseline for mathematics proficiency. Data provided by the OECD in both the 2003 and 2009 test, when mathematics literacy was the major domain assessed, showed U. S. students’ average scores were lower than the OECD average scores (Fleischman, Hopstock, Pelczar, & Shelley, 2010).

When comparing the percentage of students at each level of the PISA, a larger percentage of American students were at the lower end of the spectrum than at the high end. Figure 1 compares United States scores with OECD average scores.

Hanushek, Peterson, and Woessmann, (2010) compared the percentage of high-achieving American students with the rest of the countries who took the test in the 2006 PISA. They found only six percent of American students were high achieving. Even when scores were disaggregated by states, Massachusetts, with the highest percentage of high achieving students, had only 11.4% while 28% of Taiwanese students scored within
Researchers (Hanushek et al., 2010) compared the percentage of high achieving students who were white, or with at least one parent with a college degree. Although there has been some improvement between the 2003 and 2009 test years in the United States mathematics scores, theses scores do not come close to reaching the scores of high achieving students found in the top five countries.

Wu (2010) provided a comparison between the 2003 PISA and TIMSS mathematics sections. She described the PISA as providing information on how well students apply mathematics skills while the TIMSS provides information about how well students understand mathematics used in school. She believes these differences are due to the rationale behind each test’s construction. The PISA is constructed to provide information for policy development, and the TIMSS is developed to provide researchers with information about curriculum and instruction. In reviewing test results, the gap between Asian countries and other countries is greater on the TIMSS than the PISA.
These test differences may be due to test construction or student age related to how much schooling students had received at the time of testing.

Wu (2010) showed that American students vary greatly in their mathematics ability. Some causes may be that mathematics is not considered a meaningful activity, disengagement, or preparation gap. This gap is also evident in international assessments. International comparisons show the United States is not in the top quartile in mathematics student achievement.

Low mathematics skill puts the United States at risk in the economy of the 21st century (Bybee & Fuchs, 2006; Hanushek et al., 2010). American students need to improve their performance on both national and international tests.

American Achievement

According to the NAEP website (2012, p. 208), in 1990 the National Center for Education Statistics, which is part of the U.S. Department of Education, began giving The NAEP—commonly known as the Nation’s Report Card. The mathematics measure contains multiple choice and short answer questions where students explain their answers. The tests have questions in six areas: number properties and operations; measurement; geometry; data analysis, statistics and probability; and algebra. The test items are classified by levels of mathematical difficulty, low complexity, moderate complexity, and high complexity. The test items require students to use conceptual understanding, procedural knowledge, and problem-solving techniques while applying mathematical reasoning, connections, and communication. National Assessment of Educational Progress divides test scores into three categories: Basic with a scaled score of 214 or above, Proficient with a scaled score of 249 or above, and Advanced with a
scaled score of 282 or above in the fourth-grade. In the eighth-grade, the scaled scores are Basic with a scaled score of 262 or above, Proficient 299 or above, and advanced 333 or above.

The NAEP website also states how many students took the test and what the average scores are for each state. The scaled score national averages for fourth-graders in the last four testing years were 240 in 2009, 241 in 2011, 242 in 2013, and 240 in 2015 (National Center for Education Statistics, 2017). The national mean eighth-grade scaled score in the last four testing years were 283 in 2009, 284 in 2013, 285 in 2013, and 282 in 2015 (National Center for Education Statistics, 2017). Since 1990, when testing began, the average for both fourth- and eighth-graders has increased (Coll, France, & Taylor, 2012).

According to Closing the Achievement Gap (Williams, 2003) American schools have a large achievement gap. Minority students, Hispanics and African Americans, consistently score lower than their affluent white peers. The average scaled score of children that are eligible for the Free and Reduced Lunch Program is 226, which is equivalent to Mississippi with a ranking of 51 out of 52 states and districts. Nationally, Blacks, Hispanics, and American Indian/Alaska Natives also score at the bottom of the state averages. This trend continues in the eighth-grade. The average scaled score for students eligible for Free and Reduced Lunch Program is 269, which would rank low-income eighth-graders 50th out of 52 states and districts. Black, Hispanic, and American Indian/Alaska Native students continue to score within the bottom three states (Coll et al., 2012).
In a review of the literature, Alfinio Flores (2007) refers to this difference in achievement as preparation of opportunity gap. Students from mostly low income backgrounds, are less likely to have qualified teachers, more likely to have teachers with lower expectations, schools with less funding per student, and it is less likely that teachers will emphasize reasoning, and non-routine problem-solving (Flores, 2007; Johnson & Kritsonis, 2006). The North American Division Department of Education states that students enrolled in Adventist schools learn standards (content) and skills within a Seventh-day Adventist worldview.

Adventist Educational System

The Seventh-day Adventist Church is a Protestant church with a worldwide educational system of over 7,800 schools, colleges, and universities (Department of Education Seventh-day Adventist Church, 2016). According to a survey conducted by the National Center for Education Statistics in the 2011-2012 school year there were 800 Seventh-day Adventist elementary schools in the United States (Broughman & Swaim, 2013). According to the writers of the 2012 North American Division Mathematics Standards, they used the following resources to write the 2012 Mathematics Standards: subject-matter organization standards, state standards, CCSS, Adventist curriculum frameworks, Adventist curriculum guides/key learning, and the Adventist Education Standards (North American Division Department of Education, 2017). The 2012 Seventh-day Adventist Standards do not include Common Core-Aligned Practices. At the same time that the new standards were adopted, the North American Division Department of Education selected the Go Math curriculum for grades K – 5 or 6 and Big Ideas Math for grades 6 – 8.
The results of CognitiveGenesis, a research project sponsored by the North American Division of Seventh-day Adventist conducted from 2006 to 2009, showed that students attending Adventist schools “outperformed the national average in all subjects.” (Kido & Thayer, 2017). Seventh-day Adventist student study results in reading were reading 71st percentile rank, language arts 70th percentile rank, and mathematics was at the 62nd percentile rank. (Kido & Thayer, 2017).

This study reviewed the instructional work of teachers in the Florida Conference of Seventh-day Adventists. A five-year average of ITBS total mathematics percentiles found that the Florida Conference school average was within the median quartile. Of the 29 schools within the Florida Conference, there were eight schools in the lower quartile and seven schools in the upper quartile with a range of sixteen. These scores create a normal standard deviation. This study looked at the characteristics of eight teachers who teach at schools within the upper quartile of the Florida Conference five-year mean ITBS scores.

Effective Elementary Mathematics Instruction In this section, first I discuss what the literature says about mathematical thinking. Then, I discuss general qualities of effective instruction; general instructional strategies used to teach mathematics, such as prior knowledge, motivation, practice, feedback, emotional climate, metacognition, and student beliefs about learning. Finally, I discuss some specific mathematics instructional strategies used in effective mathematics instruction, such as conceptual understanding, vocabulary, multiple representation, direct instruction, task cognitive demand, and number sense.
Mathematical Thinking

Mathematical habits of mind first appear in the literature in a 1996 article by Al Cuoco, E. Paul Goldenberg, and June Mark (Cuoco, Goldenberg, & Mark, 1996). In this article, they proposed useful approaches for thinking about mathematics. Their general list included several skills: finding patterns, a willingness to experiment, using mathematical language to give descriptions, the ability to visualize mathematical ideas, and using evidence to make conjectures. In 2000, the NCTM developed a set of content standards and practices. The Content Standards list what students are supposed to know and the Process Standards describe how students should reason. The Process Standards include: problem solving, the ability to grapple and solve complex problems; reasoning and proof, the ability to investigate mathematical conjectures; communication, the ability to share ideas and clarify understanding; connections, the ability to view mathematics as a coherent whole; and representations, the ability to use various mathematical representations to express ideas (NCTM, 2014).

Finally, in 2009 states began adopting the Common Core State Mathematics Standards. Like the NCTM standards, the Common Core State Mathematics Standards included content standards and process standards (National Governors Association Center for Best Practices Council of Chief State School Officers, 2010b). There are eight practice standards in the CCSS. These include:

- Make sense of problems and persevere in solving them.
- Reason abstractly and quantitatively.
- Construct viable arguments and critique the reasoning of others.
- Model with mathematics.
• Use appropriate tools strategically.
• Attend to precision.
• Look for and make use of structure.
• Look for and express regularity in repeated reasoning. (National Governors Association Center for Best Practices Council of Chief State School Officers, 2010c).

Although worded differently, parallels can be seen. For students to develop these skills it is important that mathematics instructors use general and specific mathematical strategies that encourage this kind of thinking.

General Instructional Strategies

Current research on best practices indicates the following strategies are beneficial when used to teach all subjects: student application of prior knowledge, which is the students’ ability to make connections with what is known; student motivation; practice; feedback; emotional climate of learning environment; metacognition; and beliefs about intelligence and learning (Ambrose, Bridges, Lovett, DiPietro, & Norman, 2010). The following section describes these strategies in detail.

Prior Knowledge

Prior knowledge activates long-term memory and allows new knowledge to become assimilated. Prior knowledge activation has beneficial effects in learning new material (Wetzels, Kester, Merrienboer, & Broers, 2011) and is foundational knowledge that is used to make connections for new knowledge (Ambrose et al., 2010). A 2008 study of pharmaceutical students in Helsinki showed that students with more prior knowledge got a higher final grade (Hailikari, Katajaviouri, & Lindblom-YLanne, 2008).
A study conducted by Michael Pressly, et al. (1992) suggested that teachers can activate students’ prior knowledge by asking students to share with each other what they know about the subject. Prior knowledge is also activated when teachers and students develop explanations.

**Motivation**

Motivation moves and directs our actions and sets our goals. Eccles and Wigfield (1995) describe motivation as being composed of two variables: one’s expectation of success, and the value placed on the activity. Students are motivated to participate in activities that they believe will be successful, and/or activities where they view the outcomes as valuable. Therefore, students who have experienced previous positive outcomes in mathematics will be motivated to participate in future mathematics activities. Students who need to solve a mathematical problem to continue a game they enjoy, will also persevere in finding a solution.

A meta-analysis conducted by Deci, Ryan, and Koestmar (1999), listed the following as teacher strategies to improve student motivation: “(a) minimize the use of authoritarian style and pressuring, (b) acknowledge good performance without using rewards, (c) provide choice about how to do the tasks, and (d) emphasize the interesting or challenging parts of the task” (p. 15).

**Practice**

“Practice makes perfect” is a cliché often heard. According to David Sousa (2012), practice is to “repeat a motor or cognitive skill over time.” In his blog, Sousa continues to explain that, what practice does is make permanent. Material that is
practiced can later be remembered. Practice has also been called rehearsal and is defined by MacLeod (2012) as “the review of an already recoded experience in memory” (MacLeod, 2012, p. 39). Practice provides students an “opportunity to process the information through rephrasing, elaborating, and summarizing new material in order to store this material in their long-term memory” (MacLeod, 2012, p. 39). The best kinds of practice are spaced (Son & Simon, 2012) and elaborative (Benjamin & Bjork, 2000). Spaced practice or distributed practice leads to better retention than practicing in a single long block of time (Seabrook, Brown, & Solity, 2004). Elaborative practice allows for the integration of new information with information already stored in long-term memory as opposed to rote practice, which is the rote repetition of information (Benjamin & Bjork, 2000). In his book, Building Background Knowledge for Academic Achievement, Marzano (2004) describes making permanent memories as a social process that requires a minimum of about four exposures to new information in order to adequately integrate new knowledge into their background knowledge, and that these exposures must have no more than a two-day gap.

Feedback

How am I doing? Feedback is information that allows the learner to determine how well he or she is reaching a goal (Wiggins, 2012). “Helpful feedback is goal-referenced, tangible and transparent, actionable, user-friendly (specific and personalized); timely, ongoing, and consistent” (Wiggins, 2012, p. 13). Feedback does not give advice on how to proceed or provide an evaluative judgment (Wiggins, 2012). According to a lab and classroom study, “affirming or corrective feedback facilitated math-disabled students in the acquisition and retention of math facts” (Brosvic, Dihoff, Epstein, &
Cook, 2006, p. 50). A 2010 study (Gielen, Tops, Dochy, Onghena, & Smeets) examined the differences between peer feedback and teacher feedback. The study results show that peer feedback is equivalent to teacher feedback (Gielen et al., 2010).

**Emotional Climate of Learning Environment**

Researchers are not in agreement on what factors make up learning environments. In their review of the literature Cohen, McCabe, Michelli, and Pickeral (2009) list four major factors that shape and shade learning climate. The essential dimensions of learning climate are safety, teaching and learning, relationships, and environmental-structural. Safety consists of physical safety and social-emotional safety. Teaching and learning consist of four areas: quality of instruction; whether social, emotional, and ethical learning are taught; professional development; and school leadership. The dimensions of these relationships consist of respect for diversity, school-community collaboration, moral, and the connectedness of students and teacher. Finally, the environmental-structural climate consists of cleanliness; adequate space and materials; inviting aesthetic quality, and extracurricular offerings.

**Metacognition**

One definition of metacognition states that it is a “person’s declarative knowledge about the interactions between [the] person, task, and strategy characteristics” (Veenman, Hout-Wolters, & Afflerbach, 2006, p. 4). According to Paris and Winograd (1990), academic learning is improved when students become aware of their thinking as they solve school problems. “Generally, meta-cognition comprises two main components: regulation of cognition and knowledge of cognition” (Lee, Koh, Cai, & Quek, 2012, p.
When students use metacognition they use their declarative knowledge to monitor a task—generating problem-solving steps, sequencing those steps, and finally assessing outcome and re-calculating if necessary (Veenman et al., 2006).

**Student Beliefs About Intelligence and Learning**

According to Dweck (2003), children develop one of two beliefs about intelligence between the age of 10 – 12. Either intelligence is a stable “fixed trait” while other children develop a belief that intelligence is “an expandable quality” that revolves around several processes, which include effort, learning, and using effective strategies. According to two-year longitudinal research conducted by Blackwell et al, (2007), students who begin middle school with a fixed-mindset, predicted a flat grade trajectory; while students who start middle school with a malleable mindset of intelligence, predicted an upward grade trajectory.

**Specific Instructional Strategies for Teaching Mathematics**

In addition to general instructional strategies, mathematics teachers must also use specific instructional strategies. These strategies include methods that allow students to develop conceptual knowledge, the use of specific mathematical language, and guiding class discussions. Strategies also include the use of multiple representations, direct instruction of concepts, task cognitive demand, and number sense.

**Conceptual Understanding**

According to the NCTM (201400), students should learn mathematics for conceptual understanding. The NCTM provides six practices essential for teaching mathematics with teaching and learning at its core. In the NCTM (2014) Executive
Summary: *Principals to Action*, Teaching and Learning are highlighted as the core of mathematics instruction. “An excellent mathematics program requires effective teaching that engages students in meaningful learning through individual and collaborative experiences that promote their ability to make sense of mathematical ideas and reason mathematically” (2014, p. 4).

Common Core State Standards Mathematics Practices emphasize conceptual understanding for students in pre-kindergarten to 12th grades. These practices include the student’s ability to:

- make sense of problems and persevere in solving them, reason abstractly and quantitatively, construct viable arguments and critique the reasoning of others, model with mathematics, use appropriate tools strategically, attend to precision, look for and make use of structure, and look for and express regularity in repeated reasoning. (National Governors Association Center for Best Practices Council of Chief State School Officers, 2010c, p. 1)

A 2016 OECD Working Paper cites John Hatie, the Education Endowment Foundation, and Robert Marzano for attributing metacognition as an important part of teaching mathematics as well as other subjects (Echazarra, Salinas, Mendez, Denis, & Rech, 2016, pp. 12-13). Many of the MQI dimensions address student conceptual understanding and teacher strategies that reinforce student conceptual understanding of the mathematics.

**Vocabulary**

According to the Common Core Practices (National Governors Association Center for Best Practices Council of Chief State School Officers, 2010b) students in all grades should be able to construct viable and mathematically acceptable arguments using established assumptions and definitions to make conjectures. Several research studies
have shown that knowledge of content area vocabulary improves understanding of concepts (Dunston & Tyminski, 2013; Harmon, Hedrick, & Wood, 2005; Marzano, 2004; Monroe & Pendergrass, 1997; Nagy & Townsend, 2012; Smith & Angotti, 2012).

Mathematics vocabulary is different from conversational vocabulary in three ways. First mathematics is non-temporal with no past or future, only a present. Second, the language of mathematics is non-emotional. Third, communication in mathematics is very precise (Jamison, 2000). According to Nagy and Townsend (2012), “Academic language is specialized because it needs to be able to convey abstract, technical, and nuanced ideas and phenomena that are not typically examined in settings that are characterized by social and/or casual conversation” (Nagy & Townsend, 2012, p. 95).

There are two categories of mathematics vocabulary: words that are discipline specific, and words from common speech with a specialized meaning when used in mathematics.

Research in content vocabulary instruction and mathematics vocabulary instruction has consistently shown several instructional strategies found in effective instruction (Jamison, 2000; Monroe & Orme, 2002; Nagy & Townsend, 2012; Smith & Angotti, 2012). The most effective strategies include the integration of new words with prior knowledge schema, repetition, and meaningful use (Harmon et al., 2005).

Discussions

Classroom mathematics discussions are achieved when students describe, explain, defend, and/or justify their ideas about mathematics (Kosko, 2012). Using data collected for Early Childhood Longitudinal Study, which collected data from kindergarten students in the 1998 – 1999 school year through their eighth-grade school year in 2006 – 2007,
Karl Kosko (2012) found a causal relationship between daily mathematics discussion and student mathematics achievement.

Classroom discussions change the role of the teacher from dispenser and arbitrator of knowledge to the orchestrator of an environment where “students actively grapple with mathematical problems and construct their own understandings” (Stein, Engle, Smith, & Hughes, 2008, p. 4). These authors provide a framework that describes best practices in orchestrating a mathematics classroom discussion. The framework includes five practices:

- anticipating likely student responses to cognitively demanding mathematical tasks;
- monitoring student responses to the tasks during the explore phase [of the lesson];
- selecting particular students to present their mathematical responses during the discussion phase;
- purposefully sequencing the students responses that will be displayed; and
- helping the class make mathematical connections between different students responses. (Stein et al., 2008, p. 12)

**Multiple Representations**

Mathematics is created by converting the concrete world we live in into abstract representations; numbers represent quantities, other symbols are used to identify actions to be taken, finally different symbols represent unknown quantities (Kilpatrick, Swatford, & Findell, 2001; Salkind, 2007; Stein et al., 2008). A mathematical representation is defined as “external manifestations of mathematical concepts” (Pape & Tchoshanov, 2001, p. 118) According to Jerome Bruner (1964), as children grow they develop more sophisticated forms of representation; enactive representation, iconic representation, and symbolic representation. Enactive representation is through action (i.e. using manipulatives). An iconic representation is a representation through icons or visual images (i.e. using pictures, drawings, graphs, etc.). Symbolic representation is
representations through words or language (i.e. algebraic number symbols, equations, and formulas). Using Bruner’s theory, Neil Hall (1998) developed a procedural analogy theory, which sequences mathematics education from concrete, to representational, to symbolic. In addition, the Common Core Math Standards (2010) state that elementary students should be able to construct arguments using concrete referents such as objects, drawings, diagrams, and actions.

A meta-analysis of 55 studies by Carbonneau, Marley, and Selig (2012) that compared instruction with manipulatives to a control condition showed mixed results. In their review of the literature they found four theoretical explanations for using manipulatives:

(a) to support the development of abstract reasoning, (b) to stimulate learners’ real-world knowledge (c) to provide the learner with an opportunity to enact the concept for improved encoding, (d) to afford opportunities for learners to discover mathematical concepts through learner-driven exploration. (Carbonneau et al., 2012, p. 382)

The Carbonneau et al. (2012, p. 389) results show a total effect size of 0.37, while statistically significant, is a medium effect size. When disaggregated, this meta-analysis found that the benefits of using concrete manipulatives were not consistent for all learning situations. In the Carbonneau et al. (2012, p. 393) review of the literature, it would seem that the use of manipulatives in certain circumstances is the best choice teachers can make, especially when it comes to the retention of studied material.

**Direct Instruction**

Direct instruction is an instructional strategy where the teacher provides a structured, sequenced presentation of the content (Abbott, 2013). Several studies have found that direct instruction of content is an effective method of instruction (Al-
Makahleh, 2011; Darch, Carnine, & Gersten, 1984; Wilson & Sindelar, 1991). According to John Hattie’s (2009) meta-analysis the average effect size of direct instruction is 0.59. Several studies (Al-Makahleh, 2011; Flores & Kaylor, 2007; Ryder, Burton, & Silberg, 2006) found that students with special needs demonstrated significant increases in mathematical skills when direct instruction was used. In a two week study, Janicki and Peterson (2016) found that students with internal locus of control did better in small-group instruction with choice than with direct instruction.

**Task Cognitive Demand**

Mathematics instruction allows students to become proficient in solving different types of mathematical problems. Becoming proficient in mathematics involves practice with mathematical tasks. According to Wilhelm (2014, p. 636) mathematical tasks should require students to think deeply about mathematics. Doyle (1988) divides the cognitive requirement of mathematical tasks into lower tasks that require memorization of algorithms without understanding, while upper-level tasks are open, have multiple solution paths, and allow students to reason and make generalizations. Stein, Grover, and Henningsen (1996, p. 460) describe a cognitively demanding task as having two phases, first being the setup phase where the teacher explains the task to students. The second phase is called implementation, where the students work on the problem and the teacher provides scaffolding. Both Doyle (1988) and Stein et al. (1996) attest that sometimes the teacher fails to keep the cognitive demand of the problem by doing the thinking for the students and leaving them with only the algorithms to plug numbers into a formula.
Number Sense

Gersten and Chard (1999) define number sense as “a child’s fluidity and flexibility with numbers, the sense of what numbers mean, and an ability to perform mental mathematics and look at the world and make comparisons” (Gersten & Chard, 1999, pp. 19-20). Andrews and Sayers (2014) define number sense “as the ability to operate flexibly with numbers” (p. 259).

As a predictor of math achievement number sense, Jordan, Kaplan, Locuniak, and Ramineni (2007) found in their longitudinal research of 277 kindergarteners that “number sense performance in kindergarten, as well as number sense growth from the start of kindergarten through the middle of first-grade, accounted for 66% of the variance in first-grade math achievement” (p. 42). Research by Jordan, Glutting, Ramineni, and Watkins (2010) shows that a number sense test given in kindergarten has a positive correlation to third-grade math scores (p. 191).

Number sense can be improved through instruction. Research by Yang and Wu (2010) of Taiwanese third-grade students found that the group which received number sense and traditional instruction preformed significantly better when compared with the students who received only traditional mathematics instruction on tests of number sense (p. 390).

Teacher Effectiveness Teachers’ impact on academic outcomes of students has been well documented (Brophy & Good, 1984; Ding & Sherman, 2006; Kyriakides, Creemers, & Antoniou, 2009; Muijs & Reynolds, 2000). Brophy (1986) states that specific teacher behaviors increase student achievement. Linda Darling-Hammond’s (2000) review of the literature found both qualitative and quantitative studies that link
high quality in teachers with student improvement. Research has also shown that it’s the choices teachers make that impact the effectiveness of what students understand (Heck, 2009; Knoeppe, Logan, & Kieser, 2005; Stronge, Ward, & Grant, 2011).

In the last three decades, a large body of research has shown the relationship between teacher effectiveness and student growth (Darling-Hammond, 2000; Sanders & Rivers, 1996; Wayne & Youngs, 2003; Wright, Sanders, & Horn, 1997). In an analysis of 3 million student records, Saunders and Rivers (1996) found that the most important factor affecting student achievement is the “teacher effect.” This analysis, also suggests that the residual effect of teachers, either good or bad, are still measurable two years later. Saunders and River’s (1996) study of Tennessee data found a 50-percentile point difference for students who had teachers in the lowest of the highest quartile for three consecutive years. A meta-analysis of 55 studies by Carbonneau et al. (2012), compared instruction with manipulatives. They also found that the effect of teachers is both additive and cumulative.

A study of 3,000 schools and half a million students in grades three through seven, conducted jointly by Harvard and the Texas Education Department (Rivkin, Hanushek, & Kain, 2005) revealed that teacher quality accounts for at least a difference of seven and a half percent between the achievement of students. In a German study, in a sample representing different tracks in German tenth-grade classes of 181 teachers teaching 4,353 students participating in a two-year longitudinal study, Baumert et al. (2010) found that by the end of tenth-grade, a mathematics teacher’s pedagogical knowledge explained 39% of the differences in student achievement.
In a meta-analysis of research, Robert Marzano (2000) described effective schools as schools that have positive cooperation between staff, a safe school climate, consistent monitoring of student progress, curriculum that is well articulated and covered, effective use of time, supportive parental culture, strong message that academic achievement is the school’s main goal, and strong administrative.

The same meta-analysis Marzano describes the nine instructional strategies that are consistently used by effective teachers. The instructional strategies used by effective teachers include identifying similarities and differences, summarizing and note taking, reinforcing effort and providing recognition, homework and practice, nonlinguistic representations, cooperative learning, setting goals and providing feedback, generating and testing hypotheses, and activating prior knowledge.

Table 2 shows students in the 50th percentile in an average school with an average teacher will be average after two years of instruction. However, if the same students are placed in a below average school with a below average teacher, after two year the students will test in the 3rd percentile. Students placed in an effective school and teacher will be in the 96th percentile. Students in a below average school with an effective teacher will be at the 63rd percentile rank and students placed in an effective school with a below average teachers are in the 37th percentile rank. There is a 59-percentile difference in student achievement that seems to be driven by teacher effectiveness.

Identification of Effective Mathematics Teachers

Research on what makes an effective teacher has been an area of study since the 1920s (Doyle, 2016). Researchers have looked at the education, experience, and certification of teachers to unearth the characteristics of an effective teacher. According
Table 1

*School and Teacher Effectiveness: Impact on Learning Entering School at 50th Percentile After Two Years*

<table>
<thead>
<tr>
<th>Type of School and Type of Teacher</th>
<th>Percentile After Two Years</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ineffective School and Ineffective Teacher</td>
<td>3rd</td>
</tr>
<tr>
<td>Effective School and Ineffective Teacher</td>
<td>37th</td>
</tr>
<tr>
<td>Average School and Average Teacher</td>
<td>50th</td>
</tr>
<tr>
<td>Ineffective School and Effective Teacher</td>
<td>63rd</td>
</tr>
<tr>
<td>Effective School Average Teacher</td>
<td>78th</td>
</tr>
<tr>
<td>Effective School and Effective Teacher</td>
<td>96th</td>
</tr>
</tbody>
</table>

*Note.* From (Marzano, 2000)

to Doyle (2016) current research on teacher effectiveness is based on the process-product paradigm. In this “paradigm the effectiveness question is formulated in terms of relationships between measures of teacher classroom behaviors (processes) and measures of student learning outcomes (products)” (Doyle, 2016, p. 155). In this study, an effective teacher of mathematics develops student ability to apply mathematical content knowledge.

According to NCTM (2014), applying mathematical content includes solving problems, mathematically reasoning and providing proofs, communicating with others, making connections between mathematical standards, understanding and using a variety of mathematical representations. The CCSS (National Governors Association Center for
Best Practices Council of Chief State School Officers, 2010b) of Mathematical Practices also provide a description of what students can do including making sense of problems and persevering in solving them, reasoning abstractly and quantitatively, constructing viable arguments and critiquing the reasoning of others, modeling with mathematics, using appropriate tools strategically, attending to precision, and looking for and making use of mathematical structures.

Teacher Characteristics

Researchers have looked at variables in teacher education, years of experience, teacher certification, student growth in pre-and-post standardized tests, lesson artifacts (i.e. lesson plans and student assignments), and teacher observations (Gordon, Kane, & Staiger, 2006; Huang & Moon, 2009). In addition to teacher characteristics, researchers have looked at the ratings of the colleges where teachers attended, teacher certification, specialized degrees, test scores, and the courses that were taken. In a review of this literature, Wayne and Young (2003) found there was a small positive correlation between the college rating and student achievement gains. There was a small negative correlation between test scores and student achievement. When it came to degrees and courses taken, there was a significant positive correlation between teachers with degrees, extra course work, and/or standard certification in mathematics for middle school or high school and student gains in mathematics. This correlation was found only in math. Kane, Rockoff, and Staiger (2006) found that using six years of student test data when comparing certified, uncertified, and alternatively certified teachers the certification status has a very small impact on student test outcomes.
Research from the National Board for Professional Teaching Standards (NBPTS) certification has consistently shown that NBPTS certified teachers are more effective teachers (Cantrell, Fullerton, Kane, & Staiger, 2008; Goldhaber & Anthony, 2007). To become certified, teachers must pass tests and provide portfolio evidence that they have the skills listed in the National Board Standards. Research on the NBPTS certification has consistently shown that NBPTS certified are more effective teachers (Cantrell et al., 2008; Goldhaber & Anthony, 2007). To become certified, teachers must pass tests and provide portfolio evidence that they have the skills listed in the National Board Standards (NBPTS, n.d.).

Student Test Scores and Value-Added Formulas

Because teacher effectiveness is an important determiner of student success, researchers have looked to test scores to identify teachers of excellence. But tests do not account for other factors that affect student yearly growth, so value-added formulas are used to get a more accurate picture.

The use of student test scores has been used to evaluate teacher effectiveness (Lynn, 2013; Rockoff, 2004). According to Rockoff (2004, pp. 247-248) “A one-standard-deviation increase in teacher quality raises test scores by approximately 0.1 standard deviations in reading and math on nationally standardized distributions of achievement”. Ding and Sherman’s review of the literature postulates that teacher variables and student variables interact to create student learning.

Administrators and policy makers have tried to identify the characteristics that identify excellence in teaching. In the following section, I describe some of these methods. Currently, policymakers in many states have developed value-added formulas.
Value-added assessments use statistical tools to determine a teacher’s effect on student outcomes. These models try to control for factors such as demographics, socioeconomic status (SES), school, and classmates (Holley, 2008). In a review of over one million children, Chetty, Friedman, and Rockoff (2014) found that students assigned to teachers with high value-added scores were more likely to attend college, earn higher salaries, and less likely to have children as teenagers. Harvard University professor Daniel Koretz (2008) recommends that when using value-added formulas one must use specially designed tests that allow for vertical comparisons, understand that value-added formulas are an imprecise tool, and compare classes or schools that start at similar levels of performance.

**Teacher Knowledge**

Teacher characteristics are the total knowledge about mathematics and mathematics instruction, also known as pedagogy. In the teaching of mathematics this knowledge is the ability to understand mathematics and the knowledge of how students learn.

According to a report by D. L. Ball et al. (2005), teaching mathematics is a careful balance of pedagogy and mathematics. This section describes the importance of an effective teacher, and the knowledge needed to provide effective mathematics instruction.

The question of what a teacher needs to know to teach well has been a critical question for the last 40 years (Ball, 2003). No one questions that teachers need to understand the mathematics they teach, but according to Goldhaber and Brewer (1996) neither teacher certification or experience yield a statistical difference in student learning.
The same meta-analysis of the literature found that one mathematics teacher could have a statistical impact on student outcomes in mathematical knowledge. They concluded that “student achievement in technical subjects can be improved by requiring subject teaching instructional knowledge of teachers.” (Goldhaber & Brewer, 1996, p. 208).

**Pedagogical Content Knowledge**

In his groundbreaking paper, Shulman (1987) categorized teacher knowledge as a special kind of technical knowledge and described what that knowledge should be. An expert teacher’s knowledge includes knowledge of the material, knowledge of specific classroom strategies (pedagogy), knowledge of the curriculum, and knowledge of learners. According to Shulman (1987), knowledge of the material consists of understanding the subject matter. Pedagogical Content Knowledge is the ability to explain the material to students. Curricular knowledge is defined as understanding the prerequisites needed to be successful to teach the current class, understanding what students need to learn currently, and what they will be taught in the future class.

Tchoshanov (2011, p. 148) described mathematical content knowledge as being composed of three different kinds of knowledge:

- Type 1: Knowledge of Facts and Procedures
- Type 2: Knowledge of Concepts and Connections
- Type 3: Knowledge of Models and Generalizations” (p. 148).

Research found no strong correlations between student test scores and teacher Type 1 and Type 3 knowledge. However, his study revealed a strong correlation between Type 2 knowledge and student achievement.
Krauss et al. (2008) tested the content knowledge and the PCK of 189 secondary mathematics teachers in Germany. The study found that “despite high correlational interdependence, content knowledge and PCK represent different constructs” (Krauss et al., 2008). In a study of pre-service teachers in Turkey, Tumuklu and Yesildere (2007) found that although pre-service teachers need a deep understanding of mathematical content; understanding the content was not enough to become a successful teacher. Pre-service teachers also needed an understanding of pedagogy of mathematics instruction.

**Mathematical Content Knowledge**

American teachers lack a thorough understanding of mathematical content. Ma (1999) describes American teachers as lacking the connection between mathematical topics and a deep understanding of fundamental mathematics. As she compares Chinese teachers to American teachers, she describes Chinese teachers as having “knowledge packets” or a web of mathematical knowledge. She attributes American teachers’ lack of content knowledge to deficient instruction. She believes American teachers do not receive a comprehensive understanding of mathematics in elementary school, which is then transferred to what they learn in college and finally what they teach.

Ma (1999) also finds that American pre-service teachers are not given the opportunity to develop strategies that will help students think about mathematics and understand mathematical topics. This lack of PCK is not just a problem found in lower grade teachers who have taken few mathematics courses, but this lack of knowledge has also been found in teachers who have a deep understanding of the subject, and yet are weak in their understanding of how to teach the content.
**Mathematical Teacher Knowledge**

More recently, researchers have described the knowledge teachers should have when teaching mathematics. Ball, Lubienski, and Mewborn (2001) describe what a teacher must know to teach mathematics effectively. The four areas of teacher knowledge conceptualized by Ball et al. (Charalambous & Hill, 2012; Charalambous, Hill, & Mitchell, 2012) are: knowledge of the specific material, knowledge of pedagogy, knowledge of the curriculum, and knowledge of the learner (Ball, Boykin, et al., 2008). When a teacher has content knowledge, he or she understands how mathematics works. Knowledge of pedagogy implies that the teacher understands multiple ways of explaining the material to students, can make connections with prior learning, and can lead out in discussions. Curricular knowledge allows the teacher to see the depth and breadth of the curriculum; what prior knowledge students must have, what the students need to learn this year and where the curriculum taking the student. Knowledge of the student implies that the teacher can identify and/or predict common student errors and misconceptions and knows how to correct them.

In a 2005 study, Hill, Rowan, and Ball collected teacher mathematical knowledge surveys and student achievement data from 115 schools for teachers in first and third-grade classrooms. After controlling for key student and teacher covariates, such as SES and teaching experience, this study found that “teachers’ MKT positively predicted student gains in mathematics achievement during the first- and third-grades” (Hill, Rowan, & Ball, 2005, p. 399).

This research was developed into a reliable assessment of teachers’ mathematical content knowledge called MKT. The MKT is an assessment developed to assess the
specialized mathematical knowledge a teacher needs to teach effectively. Teachers’ MKT scores also correlate to student mathematics achievement, with higher scoring teachers producing students who score higher on standardized tests (Ball, Hill, & Bass, 2005). “The purpose of the MKT is to capture teachers’ mathematical knowledge for teaching” (Son & Simon, 2012). Kim (2016, p. 73) describes the reasoning needed to complete the MKT tasks stating, “the reasoning of items was shaped by elements of pedagogical context: student background, teaching purposes, and classroom artifacts, as represented in the teaching situations described in the [MKT] items” (p. 73).

The Teacher Knowledge Assessment System (TKAS) provides teachers with Computerized Adaptive Testing (CAT), which adapts to the participants’ proficiency level on the MKT. The CAT selects each question based on the participants’ previous answer. The successive questions get harder or easier depending on whether answers are correct or incorrect. Computerized Adaptive Testing continues adapting to participants’ response until the response reaches a specified level of assessment reliability. All MKT scores are normed to the population of teachers who have taken the test on the TKAS website. The scores are standardized with a score of one equaling one standard deviation above the mean. A score of 1.5 means that score is a half standard deviation higher than the average of other teachers in the TKAS. The teachers received two scores: one score for patterns, functions, and elementary algebra; and another score for number, concepts, and operations. Eligible teachers must have scored greater than zero in at least one of the assessments to be included in this study.
Observation of Instruction

Another way to assess teacher effectiveness is by observations of teachers in the act of teaching. In order to standardize observation of instruction, several observation protocols have been developed. Measure of Effective Teaching (MET) research by Kane and Staiger (2012) reviewed five observational protocols; Framework for Teaching, Classroom Assessment Scoring System, Protocol for Language Arts Teaching Observations, MQI, and UTeach Teacher Observation Protocol. Each observation rubric was evaluated for reliability and student outcomes. The combined measures identified teachers with high student gains. This research listed three main implications:

1. High levels of reliability of classroom observations require several quality assurances: Observer training and certification; system-level “audits” using a second set of impartial observers; and use of multiple observations whenever stakes are high.

2. Evaluation systems should include multiple measures, not just observations or value-added alone.

3. The true promise of classroom observations is the potential to identify strengths and address specific weaknesses in teachers’ practice (Kane & Staiger, 2012, pp. 13-14).

Mathematical Quality of Instruction

In this research, I used the MQI rubric. The MQI is a standardized observation tool designed to assess the dimensions that comprise the quality of mathematical instruction (Gielen et al., 2010). A preliminary report using the MQI, the MET Project (Bill & Melinda Gates Foundation, 2012), a partnership of over 3,000 teachers and volunteers who are helping to identify what great teaching looks like, has shown a
positive correlation between MQI observation scores and student achievement scores.

The MQI reviews four dimensions of mathematics instruction. Each dimension has a rubric describing not present, low, medium, or high teacher and student actions. The MQI is designed to be used with videotaped lessons, not live presentations. The class period is then divided into segments of 7.5 to 3.5 minutes. Segments are viewed and coded with the rubric.

Development of the MQI

The MQI, developed by Hill in collaboration with Harvard University and the University of Michigan, reliably measures the mathematical work during mathematics instruction (MET Project, 2010). The MQI was developed to identify the quality of mathematics instruction within the classroom (National Center for Teacher Effectiveness [NCTE], 2011).

The theoretical framework of the MQI is based on the belief that teaching is a synergistic endeavor that is greater than the sum of its parts (Cohen, Raudenbush, & Ball, 2003). According to Cohen et al. (2003), the learning environment is created by the relationships between the teacher and the content, the teacher and the student, the student and the content, and between and among students. Each relationship provides one piece of the educational puzzle. The MQI uses these relationships when describing the act of teaching mathematics. The relationships are described as “Instruction as Interactions.” This can be seen in Figure 2.

Next, the MQI developers looked at other teacher observation rubrics in the 1990’s and found that these instruments focused on specific types of instruction (LMT Project, 2011). The authors wanted an instrument that addressed mathematics instruction...
regardless of pedagogy or instructional topic. The MQI was designed to identify “interactions involving teachers, students and contents…[looking at both] the distinctive character…[and its] rank, level or grade” (LMT Project, 2011, p. 30).

The authors videotaped nine lessons from ten teachers and conducted interviews. They developed codes as an iterative process, first making broad distinctions, then engaging in fine-grained observations, bringing in examples from the literature, and then back to broad distinctions to start the cycle again (LMT Project, 2011, p. 32).

Mathematical Quality of Instruction developers noted that appropriate mathematical instructional strategies are critical to helping students become proficient. This view is corroborated by other writers, according to Common Core Mathematics Practices (National Governors Association Center for Best Practices Council of Chief State School Officers, 2010b).
Students at all levels must become skilled at making sense of problems and persevering in solving them. Students must be able to reason quantitatively, construct their own viable arguments and critique the reasoning of others. They should have the ability to model mathematics and strategically use appropriate tools to solve problems. Mathematically proficient students attend to precision, find pattern or structure in mathematical problems, and they discover and express regularity in repeated reasoning (National Governors Association Center for Best Practices Council of Chief State School Officers, 2010b). Educators must use specific strategies so students will become proficient when applying mathematical thinking practices.

**MQI Instructional Strategies**

Table 3 provides a list of the MQI instructional strategies and their relationships with “Instruction as Interaction.” The MQI provides for the observation of 16 instructional strategies. The instructional strategies included in the MQI can be divided into three categories or relationships: teacher to content, teacher to student, and student to content. Within each relationship several instructional strategies were observed when using the MQI.

**Correlation MQI and MKT**

Several studies (Hill et al., 2008; Hill, Charalambous, Hill, & Mitchell, 2012) show that MQI scores correlate with MKT scores. The MKT uses the same theoretical framework as the MQI. Research done by the MKT developers found that teachers improved the instructional ability and their MKT scores when the teachers used material
Table 3

**MQI Relationships**

<table>
<thead>
<tr>
<th>Teacher to Content</th>
<th>Teacher to Student</th>
<th>Student to Content</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linking Between Representations</td>
<td>Remediation of Student Error</td>
<td>Students Provide Explanations</td>
</tr>
<tr>
<td>Explanations</td>
<td>Teacher Uses Student Mathematical Contribution</td>
<td>Student Mathematical Questioning and Reasoning</td>
</tr>
<tr>
<td>Mathematical Sense-Making</td>
<td>Patterns and Generalizations</td>
<td>Student Communicate about the Mathematics of the Segment</td>
</tr>
<tr>
<td>Multiple Procedures or Solution Methods</td>
<td>Mathematical Language</td>
<td>Task Cognitive Demand</td>
</tr>
<tr>
<td>Patterns and Generalizations</td>
<td>Mathematical Content Errors</td>
<td>Students Work with Contextualized Problems</td>
</tr>
<tr>
<td>Mathematical Language</td>
<td>Imprecision in Language or Notation</td>
<td></td>
</tr>
<tr>
<td>Lack of Clarity in Presentation of Mathematical Content</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*Note.* From (Boston, Bostic, & Lesseig, 2015)

and positive correlation between the two measures (Hill et al., 2008, p. 430).

**Other Research Use of the MQI**

An article by Boston et al. (2015) compared mathematics observational tools. They reviewed the Reform-Oriented Teaching Observation Protocol, the Instructional Quality Assessment in Mathematics; and the MQI. This article reviewed each observation protocol and found that each protocol provided different windows on mathematics instruction. They listed the uses of the MQI as professional development, curriculum
provided by the researchers to teach mathematics. A study comparing the MKT and MQI showed a strong implementation, pre-service teacher education, and large-scale assessments of mathematics teaching (Turnuklu & Yesildere, 2007).

A study funded by the Bill and Melinda Gates Foundation for the MET (2013) project using the MQI Lite, found that teachers in the study were likely to be teaching mathematics and quite unlikely to make mathematical errors.

Mathematics lessons were scored very low for three of the competencies: working with students and mathematics, the richness of mathematics, and student participation in meaning making and reasoning. For these competencies, about 1% of lessons achieved the highest rating and more than 70 percent of lessons received the lowest rating. (Bill & Melinda Gates Foundation, 2013, p. 166)

A doctoral dissertation by Krista Lynn Strand (2016) used the MQI rubric to direct teacher professional development. She found that when teachers understood the MQI rubric they were better able to identify student use of Common Core Practices (National Governors Association Center for Best Practices Council of Chief State School Officers, 2010c). Another dissertation by Katherine Marin (2015) found that a small sample of teachers receiving professional development using the MQI rubrics and videos allowed teachers to better assess their ability, to plan lessons, and reflect on student engagement in Common Core Practices.

Summary

The purpose of this literature review was first to provide the reader an understanding of the challenges faced by the educational systems in the United States when it comes to teaching mathematics, then to identify instructional strategies that good teachers employ that contribute to student success. Finally, this literature review
discusses teacher excellence and protocols used to identify it. This research lays the foundation for understanding the status of American mathematics education, the ways that teachers of excellence provide instruction in the United States, and how these teachers are identified.
CHAPTER 3

METHODOLOGY

Introduction

The goal of this study was to identify schools with students whose five-year average math scores were at or above the 50% percentile, and observe their mathematics teachers to describe how those teachers deliver their mathematics instruction. This chapter is organized into four primary sections. In the first section, I discuss the research design, myself as the primary research instrument, and the purposeful sampling process. In the second section, I discuss data collection methods. And in the third section, I describe data analysis methods. The fourth section is concerned with trustworthiness and generalizability. The key question that shaped the research design was: In what ways do teachers of high achieving mathematics students deliver mathematics instruction?

Research Design

A case study design was chosen because it provides a method whereby the research questions can be answered (Rowley, 2002). A case study is “an empirical inquiry [method] that investigates a contemporary phenomenon in depth and within its real-life context” (Yin, 2009, p. 18). Grünbaum (2007) lists seven characteristics of case studies, which include

the identification of the natural environment, a natural phenomenon to be studied, the researcher has no control of the properties studied, case studies use multiple data sources, case studies identify the surrounding details, the
phenomenon is described thickly, and the researcher uses holistic analysis. (Grönbaum, 2007, pp. 79, 82)

My research design fulfills all seven case study characteristics identified by Grunbaum (Grönbaum, 2007). I observed teachers in their classrooms doing what they normally do to teach mathematics. Therefore, I did not have control of any of the study events. I had multiple video data sources. Video data was collected in three schools from a total of eight teachers. The videos of each lesson were divided into 7.5-minute instructional segments. These data sources allowed me to describe the phenomenon richly and thickly using a holistic analysis.

This research describes teachers found to be effective because of their knowledge of teaching mathematics as identified on the MKT, and who teach in schools that have a five-year average ITBS score above the Florida Conference mean. In my analysis, I looked at teacher knowledge of specific mathematics content, knowledge of instructional strategies, knowledge of curriculum, and knowledge of the learner through the prism of the MQI rubric. The aim of this research was to study the characteristics of teachers and thereby bring about an “awareness and understanding of the direct experience of excellent mathematics teachers” (Bentz & Rehorick, 2009). I attempted to understand the “logic, and interrelationships that are contained in the phenomenon under investigation” (Conklin, 2007, p. 277). The data collected became the text, which I have interpreted into generalities and themes.

Yin (2009) and Shen (2010) identify the concerns that case studies are prone to researcher bias and are not generalizable. However, Flyvbjerg (2011) argues that subjectivity and bias are fair criticisms of all types of research. He makes the assumption that all expert learning consists of experience with many cases, therefore case study
design is “central to human learning” (Flyvbjerg, 2011, p. 303). Case studies provide opportunities for advanced learning of the phenomenon being studied.

This study is a multiple case study and follows a design suggested by Yin (2009). Figure 3 describes the process I used to organize this study. The first step included developing the research question, identifying cases, and choosing collection protocol. Step two consisted of videotaping cases and writing individual case reports. Finally, step three entailed writing cross case reports, and the development of conclusions and explanations.

![Figure 3. Case Study Method. Adapted from (Yin, 2009, Ch 2, Sec 3, Fig 2.5).](image)

**Figure 3.** Case Study Method. Adapted from (Yin, 2009, Ch 2, Sec 3, Fig 2.5).

**Researcher as a Primary Instrument of Research**

A qualitative researcher is part and parcel of the research process (Becker, 1998; Merriam, 1988; Rajendran, 2001). Many critics claim that qualitative research is subjective and biased (Rajendran, 2001). Becker (1998) quotes Thomas Kuhn who
describes the qualitative observation and analysis process as shaped by the researcher’s ideas, beliefs, and perceptions.

I come to this research from two positions, first as a student who has struggled understanding mathematics, and second as an educator with 38 years of experience teaching mathematics to students. I have held three positions in the Florida Conference of Seventh-day Adventists, first as special education teacher, then as vice-principal, and finally as a first-grade teacher. I have worked with Florida Conference teachers providing professional development in mathematics to build capacity before the implementation of the Go Math curriculum.

Although I have worked to provide teacher growth and development in mathematics capacity, in my current role as a first-grade teacher, none of the Florida Conference teachers report directly to me. My experiences as a student, a teacher, and as a researcher allow me to see and describe the story of excellence in mathematics education through the lenses of my experiences.

In preparation for my research, I trained in the usage of the MKT and MQI. I attended MKT training provided by the Education Department of the University of Michigan at Ann Arbor. I also requested and got permission from the NCTE at Harvard University for online training webinars on using the MQI system. The webinars consisted of sixteen hours of training including six modules: two for Richness of the Mathematics, and one for each of the other MQI dimensions; Working with Students and Mathematics, Errors and Imprecision, Common Core-Aligned Student Practices, and Whole Lesson Codes. Each module consisted of an explanation of the meaning of each code, how each code is scored, and several videos to practice coding with explanations for each score.
given. I watched the modules multiple times at intervals of three to six months to immerse myself in the protocol framework.

**Purposeful Sampling**

The goal of the sample chosen for the research was to provide the study with a rich and varied picture of the subjects chosen. Merriam states, that “purposive sampling is based on the assumption that one wants to discover, understand, gain insight, [from particular cases] therefore one needs to select a sample from which one can learn the most” (Merriam, 1988, p. 48). Onwuegbuzie and Leech (2007) identify three types of sampling schemes, “external statistical generalizations which rely on random or probabilistic samplings. Analytic generalization is data analysis, which generalizes the words and actions of a given population and case-to-case transfer which involves making generalizations from one case to another (Onwuegbuzie & Leech, 2007, p. 240). I used cases to develop analytical case-to-case transfer as a way of understanding what strategies this group of teachers used and which strategies they did not use. In the 2016 – 2017 school year, the Florida Conference of Seventh-day Adventist had twenty-nine schools throughout the state of Florida. There were two one-teacher schools, 20 schools where each teacher had between two and four grades, and seven schools where each teacher had single grade classes.

This purposeful sample consisted of elementary school mathematics teachers in the Florida Conference of Seventh-day Adventists whose school’s five-year average NCE scores on the mathematics subtest of the ITBS (Hoover, Dunbar, & Frisbie, 2007) were at or above 50th NCE. The Florida Conference average on the ITBS between 2008 through
2012 fall testing was 47th NCE with a standard deviation of 4.9. There were 10 schools at or above the 50th NCE.

The teachers eligible for my study were those whose score on the MKT was equal to or greater than the mean. I used a computerized version of the MKT found on the TKAS (LMT Project, 2009) website. The assessments were taken online by Florida Conference first- through fourth-grade teachers collectively in May of 2012. Of the teachers who took the test on May 30, 2012, there were 25 teachers who received a score of zero or above on one of the assessments. Teachers in grades six through eight were asked to take the test individually after the schools were identified. Distance also contributed to my sample. I only chose schools that were within a 200-mile radius of Orlando, Florida area because that was the distance I could travel in one day.

A total of 10 teachers met the following criteria: MKT’s score above the mean, their school’s ITBS average greater than 50th NCE, and within a 200-mile radius of Orlando. One teacher did not wish to be videotaped and another teacher could not get parental consent to video record their children. Of the nine teachers left, the single room school teacher was videotaped, but the instruction was more similar to tutoring students; this special case was not included in this study. That left a total of eight teachers to be included in this study.

I chose a balance of teachers in grades one through eight. My sample consisted of two teachers who taught first-grade, a first- and second-grade teacher, a third-grade teacher, a fourth-grade teacher, a fifth-grade teacher, and two middle school teachers.
Data Collection

As an investigation tool, Merriam (1988) states that observations provide researchers an opportunity to gather data in the natural setting of the phenomenon. Merriam argues that a researcher might see what has become commonplace to participants. She quotes Reinharz (1979) who “suggests that the social setting is rarely disrupted by the presence of an observer” (p. 96).

Merriam notes the concerns of critics that the observer may taint or “affect what is being observed” (Merriam, 1988, p. 95). She further quotes critics, who believe that observations are highly subjective and therefore not reliable. She postulates that observer training mitigates the “inaccuracies of spontaneous observations by an untrained observer” (Merriam, 1988, p. 88).

One way to assess teacher effectiveness is through direct observation (Mangiante, 2011). Valid teacher observations require two components, a valid form and a trained observer (Goe, Holdheide, & Miller, 2011). To fulfill these requirements, I completed training provided by the NCTE at Harvard University Center for Education Policy Research on the use of the MQI rubric. I became an observer where my presence was known to the class and teacher, but my participation in the group was secondary to my “role of gathering information” (Merriam, 1988, p. 93).

The data collected for this study provided the context and the reality of teachers whose students show exemplary growth. I videotaped eight teachers in three schools. A total of 14 lessons were observed. These lessons were divided into 80 segments, between 3.5 and 7.5-minutes in length.
In this research, I taped a total of five lessons in single grade classrooms in first-through second-grade classrooms, five lessons in single grade classrooms third- through fifth-grade, a lesson in a single sixth-grade, a lesson in an eighth-grade classroom, a lesson in an eighth-grade classroom where students were divided into two levels and were working in separate lessons, and a lesson in a seventh-grade classroom.

I also taped one lesson in a multi-grade classroom where the teacher had a total of five students in five grades. I did not use his data in this research because I considered it an outlier. The multi-grade teacher provided instruction at the teacher’s desk. The instruction consisted of providing students with detailed procedural explanations and examples. During the lesson only four of the rubric criteria were present: use of class time, use of mathematical language, student remediation, and lack of teacher errors or imprecision. The other dimensions were coded as not present. This mathematics class seemed more like a tutoring session where each student was provided with help exactly where he or she needed, but both remediation and instruction were procedural.

**Data Analysis**

**Procedures**

Case study data requires a detailed write-up of the qualitative, and in some instances, quantitative data for each case with general descriptions, tabular displays or graphs of information, and narrative transcripts of interviews (Eisenhardt, 1989). This analysis consists of finding common categories within each case. Next, I used cross-case pattern searching, by selecting pairs of cases and then listing the similarities and difference between each pair. “This tactic forces the researcher to look for the subtle similarities and differences between cases . . . go beyond initial imprecision through the
use of a structured and diverse lenses on the data” (Eisenhardt, 1989, pp. 540-541). As suggested by Miles and Huberman (1984), I summarize my findings using arrays or matrices of categories in a cross-case analysis.

Description of the MQI Rubric

The MQI protocol has 16 lesson codes grouped around four dimensions. In addition, each dimension has an Overall Dimension code and there are ten Whole Lesson codes. I did not use the Overall Dimension codes or the Whole Lesson holistic codes. To understand what effective teachers are doing, I only focused on the sixteen lesson codes. I used the instructional triangle relationship, with the MQI dividing mathematics instruction into four instructional dimensions: Richness of the Mathematics, Working with Students and Mathematics, Teacher Errors and Imprecisions, and Common Core-Aligned Student Practices. The Richness of the Mathematics strategies focus on teacher activities related to mathematical content. These activities include information the teacher provides, class discussion of ideas, mathematical connection and explanations, the use of multiple procedures and generalizations, and mathematical language. The Working with Student Mathematics dimension codes how well teachers respond to students’ mathematical needs. This dimension codes teacher remediation of student errors or difficulties and how teachers respond to student mathematical productions. This dimension identifies the relationship between the teacher and the students’ side of the triangle. The student participation and meaning-making and reasoning section correlates to the Common Core Math Practices. This dimension codes the relationship between students and the content. The MQI also provides a dimension that codes for teacher mistakes. Errors and Imprecision codes mistakes teachers make in explaining and
notating mathematics. This dimension identifies a relationship between the teacher and the content. Each dimension allows the observer to mark a rubric describing teacher actions during instruction.

Teacher actions are coded as not present, low, mid, or high with a description given for each code. A high score is a positive mark in all except the Errors and Imprecisions codes, where high scores are negative marks. Each segment was coded as N for not present during segment, L for low, M for medium, or H for high, for each dimension of the MQI rubric. Error! Reference source not found. provides a detailed description of each dimension and strategies within each dimension.

Following the training, I used the MQI rubric as I analyzed the instruction on the taped lessons. I watched the whole lesson and coded segments that were between 3.5 to 7.5 minutes long.

The 80 videotaped instructional segments gathered during data collection were reviewed in two ways. First, I viewed each lesson as an individual case and provided accurate and rich descriptions of classroom activities, coding the lesson, and finally, adding notes to the codes as examples for each code given. Secondly, I looked at the cases collectively. My reason for collectively analyzing the data stems from my observation that teachers spend one lesson or part of a lesson focusing on a concept that provides for great richness in the mathematics instruction, while another lesson may provide student practice to understood concepts or to develop fluency. By grouping all the segments, I felt that it was possible to get a better and clearer picture of what happens during math instruction of effective teachers. The teachers provided a total of 80 observational segments of mathematics instruction. I compared each code across the
<table>
<thead>
<tr>
<th>Table 4</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Description of the MQI Dimensions and Strategies</strong></td>
</tr>
<tr>
<td><strong>Classroom Work is Connected to Mathematics</strong></td>
</tr>
<tr>
<td><strong>MEANING OF FACTS AND PROCEDURES</strong> – Meaning includes why facts are true, why procedures work, making sense of quantities, definitions, the relationship between numbers, or why solutions make sense</td>
</tr>
<tr>
<td><strong>Linking Between Representations</strong></td>
</tr>
<tr>
<td><strong>Explanations</strong></td>
</tr>
<tr>
<td><strong>Mathematical Sense-Making</strong></td>
</tr>
<tr>
<td><strong>MATHEMATICAL PRACTICES</strong> – This section includes examining and comparing solutions methods, making mathematical generalizations, and using precise language.</td>
</tr>
<tr>
<td><strong>Multiple Procedures or Solution Methods</strong></td>
</tr>
</tbody>
</table>
### Patterns and Generalizations
This code identifies teacher or student(s’) deductive reasoning. Teacher and student(s’) notice a pattern or make a generalization about mathematical ideas or when teacher and/or student(s) create a mathematical definition based on examples. A score of high means that the pattern or generalization was not only noticed but the given explanation was detailed, clear, and complete.

### Mathematical Language
This code captures the fluency of mathematical language use in the classroom by teacher or students and whether the teacher encourages and supports the use of mathematical language. A score of high means that the use of mathematical language was correct and dense during the segment, or there was explicit instruction on the use of mathematical language during the segment, and instances when students use terminology correctly when describing ideas.

### WORKING WITH STUDENTS AND MATHEMATICS – The extent that the teacher understands and responds to student mathematical contributions or errors.

### Remediation of Student Errors and Difficulties
This code identifies teacher decisions regarding student error or misunderstanding. Remediation can be either procedural, which helps the student correct his/her mistake but does not address what the answer means; or conceptual remediation where the teacher addresses the error and explains why this is a problem. In addition, this code identifies when a teacher anticipates student error and provides instruction. A code of high is given when remediation is conceptual in nature discussing with student(s) the causes of the error, or the teacher explicitly explains that he or she anticipates a certain error; thereby helping the students avoid it.

### Teacher Uses Student Mathematical Contributions
This code identifies how the teacher uses student statements to keep the lesson moving. Student contributions can include but are limited to: answers to questions, comments of mathematical ideas, explanations, representations, generalization, questions to the teachers, and student work. A score of high is used when student contributions are an integral part of the lesson development and are more than pro-forma statements answering closed questions.
### Table 4—Continued

**ERRORS AND IMPRECISION** – Captures teacher errors or imprecision in language and notation, or lack of clarity or precision in the teacher’s presentation of the content.

<table>
<thead>
<tr>
<th>Description</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mathematical Content Errors</strong></td>
<td>This code captures mistakes in the content of what is being taught by the teacher, for example, solving a problem incorrectly or forgetting a condition in a definition. A high score would mean that the teacher makes content errors throughout the segment.</td>
</tr>
<tr>
<td><strong>Imprecision in Language or Notations</strong></td>
<td>This code covers mistakes in the use of mathematical symbols, in the use of mathematical language, or general language. A high score would mean that one or more of these mistakes occur throughout most of the segment.</td>
</tr>
<tr>
<td><strong>Lack of Clarity in Presentation of the Mathematical Content</strong></td>
<td>This code points out when the teacher explanations of the mathematical concept lacks clarity or is confusing. A high score would mean most the segment contained this error.</td>
</tr>
</tbody>
</table>

**COMMON CORE-ALIGNED STUDENT PRACTICES** – Students engagement in mathematics is evident through their mathematical explanations, conjecturing, justifying of ideas, or pattern-recognition. Tasks students are engaged in, are cognitively demanding such as defending their mathematical claims, looking for patterns, generalizing, and conjectures, or working on complex or non-routine problems.

<table>
<thead>
<tr>
<th>Description</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Students Provide Explanations</strong></td>
<td>This code identifies when students provide explanations as to why a mathematical idea works. This explanation can be correct or incorrect, constructed alone or with the teacher, or complete or incomplete. The main point here is that students are trying to make sense of the plan. A high score means that the segment is characterized by student explanations.</td>
</tr>
<tr>
<td><strong>Student Mathematical Questioning and Reasoning</strong></td>
<td>This code is meant to capture other student thoughts about the mathematics that are not an explanation. For example, if a student provides a justification why the student disagrees with a claim. Student reasoning characterizes a high scoring segment.</td>
</tr>
</tbody>
</table>
Table 4—Continued

<table>
<thead>
<tr>
<th>Code</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Student Communicate about the Mathematics of the Segment</td>
<td>This code determines how significant students’ comments are. Any student comment, which was coded in explanations and mathematical questioning, would count here as well. In addition, solution methods are counted in this code as written communication such as solving problems at the board. In this code, brief single word answers to teacher questions are coded as low, while segments that are more complete descriptions of student thinking are coded as high when the segment is characterized by student comments.</td>
</tr>
<tr>
<td>Task Cognitive Demand</td>
<td>This code looks at how deeply students are thinking or are engaged by the mathematics work and the complexity of the activities or problems. The segment is coded as high when the students are actively engaged in thinking for themselves.</td>
</tr>
<tr>
<td>Student Work with Contextualized Problems</td>
<td>This code captures the student work with contextualized problems focusing on who is doing the thinking, the teacher or the students. Segments are coded as high when the students have the opportunity to and reason about contextualized problems.</td>
</tr>
</tbody>
</table>

*Note. From (MET Project, 2010; NCTE & Harvard University, 2012)*
teachers looking for commonalities and differences within the codes. Finally, I reviewed the data within grade clusters of first - second, third - fifth, and sixth - eighth. I looked for features that grades had in common, as well as differences between grade clusters.

Two rubric elements are not listed in the results chapter because there was limited evidence of them: Use of class time and Errors and Imprecision. Of the 80 video segments, only one segment did not have the teacher using the full class time for math instruction. There were only three mistakes found in the 80 video segments. One teacher did not catch a student’s computational error. One segment showed the teacher using imprecise language. The third segment had a teacher making a mathematical error when describing a fraction. Because the teachers were chosen from a pool of teachers who scored at least average on the MKT, it appears they understand the mathematics they are teaching and do not make mathematics errors.

**Trustworthiness**

Reliability of the MQI

Hill et al. (2012) analyzed three studies where a total of 76 raters used the MQI to view 290 teachers teaching three lessons per teacher in two studies, and six lessons per teacher in another study. This analysis found that the MQI is sensitive to rater quality, therefore, I completed the training provided by Harvard University and the NCTE (2012). Hill et al. (2012) also recommend that two raters view three to four lessons for high stakes decisions. High stakes decisions include hiring and firing of teachers. Because my research was not high stakes and because the training was difficult and time-consuming, I was unable to engage another rater for my study. However, by increasing the number of
observations, one increases the $p$ value of the total (Error! Reference source not found.). I observed several
Table 5

Comparison of Reliability Estimates (p): Whole Lessons versus First Thirty Minutes

<table>
<thead>
<tr>
<th>Number of Raters</th>
<th>Richness</th>
<th></th>
<th>Errors and Imprecision</th>
<th></th>
<th>Student Participation in Meaning-Making and Reasoning</th>
<th></th>
<th>Working with Student and Mathematics</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Whole Lesson</td>
<td>30 Min</td>
<td>Whole Lesson</td>
<td>30 Min</td>
<td>Whole Lesson</td>
<td>30 Min</td>
<td>Whole Lesson</td>
</tr>
<tr>
<td>One Lesson</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>One Rater</td>
<td>0.45</td>
<td>0.50</td>
<td>0.37</td>
<td>0.34</td>
<td>0.46</td>
<td>0.32</td>
<td>0.29</td>
</tr>
<tr>
<td>Two Rater</td>
<td>0.58</td>
<td>0.59</td>
<td>0.50</td>
<td>0.46</td>
<td>0.59</td>
<td>0.41</td>
<td>0.41</td>
</tr>
<tr>
<td>Three Rater</td>
<td>0.64</td>
<td>0.63</td>
<td>0.57</td>
<td>0.53</td>
<td>0.65</td>
<td>0.45</td>
<td>0.48</td>
</tr>
<tr>
<td>Four Rater</td>
<td>0.67</td>
<td>0.65</td>
<td>0.61</td>
<td>0.57</td>
<td>0.68</td>
<td>0.48</td>
<td>0.53</td>
</tr>
<tr>
<td>Two Lessons</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>One Rater</td>
<td>0.56</td>
<td>0.65</td>
<td>0.51</td>
<td>0.49</td>
<td>0.63</td>
<td>0.49</td>
<td>0.41</td>
</tr>
<tr>
<td>Two Raters</td>
<td>0.71</td>
<td>0.73</td>
<td>0.64</td>
<td>0.62</td>
<td>0.74</td>
<td>0.58</td>
<td>0.55</td>
</tr>
<tr>
<td>Three Raters</td>
<td>0.76</td>
<td>0.77</td>
<td>0.71</td>
<td>0.68</td>
<td>0.79</td>
<td>0.62</td>
<td>0.62</td>
</tr>
</tbody>
</table>

*Note.* Adapted from (Hill et al., 2012, p. 13).
teachers more than once. In a presentation given to NCTE (2011) Hill stated that increasing the number of raters or the number of observations is interchangeable to get the same \( p \) value. Although it is possible that these video-taped lessons are different from the teachers’ general instruction, the teachers did not have any incentive to select lessons strategically as no rewards or inducements were involved with the collection of data.

**Triangulation of Research**

According to Yin (2009), it is beneficial to use multiple sources of evidence, because they provide “converging lines of inquiry” (Yin, 2009, p. 47), which aid in the process of triangulation and corroboration. My triangulation occurs within and across grade levels of first through second-grades, third through fifth-grades, and sixth through eighth-grades. I also compared two types of data: thick descriptions and the rubric results.

**Generalizability**

This research is generalizable by its readers because I attempted to provide descriptions that allow the reader to share in my observations. Stake (1978) describes naturalistic generalizations as knowledge that “develops within a person as a product of experience” (Stake, 1978, p. 6). According to Yin (1994), one of the strengths of case studies is that they provide rich descriptions, which allows the reader to experience the situation. I provided details using multiple reviews of the observations to make a thick and robust picture of the participants.

**Issues of Ethics**

Ethical issues such as informed consent, the relationship between participant and researcher, potential harm to participants, and confidentiality were addressed during the
Institutional Review Board (IRB) process at Andrews University. My study was approved by the Andrews University IRB committee.

Summary

The methods used in this study allowed me to identify characteristics of effective teachers. First though the scores teachers obtained on the MKT and then through their actual teaching strategies using the MQI. By finding commonalities in what these teachers do, my study provides a blueprint for professional development of in-service teachers and the education of pre-service teachers.
CHAPTER 4

VIEWS FROM WITHIN THE MATHEMATICS
CLASSROOM

Introduction

This chapter describes the methods of mathematics instruction within the classrooms of teachers who were identified as effective teachers by the MKT and their school IBTS normal curve equivalent scores. These narratives of instruction will allow us to peek inside the pedagogical instructional world of eight teachers during fourteen separate lessons. At the end of the narrative the lessons are viewed through the lens of the MQI. The MQI codes provide descriptions of important dimensions in quality mathematics instruction. Looking at lessons using descriptive lenses and MQI lenses allows me to provide a rich picture of effective teaching.

Observation One - Grade One

I visited Jonatha for the first time in the spring of 2014. Her first-grade class was composed mostly of boys, and she used a variety of strategies to keep them focused on their work. Math class started with fluency work. Using addition and subtraction flash cards. Jonatha reminds the students that this time she is using addition and subtraction facts. As Jonatha reads and flashes math facts, students who think they know the answer stand behind their chairs. Jonatha calls on the first student who stands. Before starting she reminds the students not to call out any answers. While she flashes the card she also asks
students to generalize about what strategies can be used to answer the math facts. When
she flashes $4 + 4$, she asks the class what type of problem is this. The students’ chorus,
“A doubles problem.” She keeps the student’s attention by asking that some of the math
facts are just for girls or students with blue uniform shirts.

When one student gives the wrong answer, Jonatha encourages her and provides a
strategy for the student to solve the problem, “You are very, very close, count on to
figure it out.” The student then provides the correct answer and the other students cheer
her on. Whenever a student makes a mistake the teacher gives them the opportunity to
correct their answer by giving them a solution strategy. When one student couldn’t use
the strategy to figure out the answer, Jonatha asked the rest of the class to help him out by
using the count on strategy. She asks, “What number do we start with?” They respond,
“The bigger one.” Then they proceed to count on, “8, 9, 10, 11, 12, 13.” Jonatha uses her
fingers to track that the students have counted on five more. Next, she tells the students
that the next problem will be a doubles problem.

She flashes $8 + 8$ for all children to chorus. Several students answer 18. Jonatha
fluidly puts down the math flash cards and begins clapping a rhythm. The students seem
to know the rhythm and follow suit. She and the students begin to chant a poem posted on
the white board:

“It’s the Doubles! Let’s go! Let’s go!

It’s the Doubles, and we start with zero:

$0 + 0 = 0$     Oh!

$1 + 1 = 2$     Oooh!

$2 + 2 = 4$     More
3 + 3 = 6   Kicks!
4 + 4 = 8   Great!
5 + 5 = 10  And then…
6 + 6 = 12  Dig and delve!
7 + 7 = 14  Jellybean!
8 + 8 = 16  You’re a queen!
9 + 9 = 18  Let’s lean!
10 + 10 = 20 That’s Plenty!

Here we go again!”

Then they repeat the chant while facing the back of the classroom without looking at the chart. Eric interrupts the beginning of the chant and Jonatha allows him. He points out that there are no answers with the numbers eleven, thirteen, or fifteen. Jonatha praises him, “That’s great you found a pattern.” She extends that idea. “Remember when we count 2, 4, 6, 8, 10. What kind of pattern are you counting by? What do we call those?” Eric answers, “Even numbers.” Then they continue chanting.

When the chant is over the students move to the carpet area. Jonatha connects the lesson to her three-week-old baby nephew. She wants to buy him three gifts and she’s trying to figure out if she has enough money. She wants to buy a hat for eight dollars, a toy for four dollars, and socks for three dollars. She asks the students for ideas on how to figure out how much money she needs. Ralph says, “You need four plus three, that’s seven dollars.” Kenny shouts out, “You need 15 dollars.” Enrique shouts out, “That’s more than one problem.” Jonatha signals the eager students to wait, allowing Hector a
chance to complete his thought. Hector continues, “Hey wait a minute that’s a doubles-plus-one problem. The answer is 15.”

Jonatha then restates Hector’s thinking using linking cubes, “I think you first said to add three plus four?” Hector agrees. Jonatha continues snapping three cubes of one color with four cubes of another color, matching the cubes with the numbers. “What did you do next?” Jonatha continues, “So you added the seven and then you added the eight.” Hector agrees, “That’s 15”.

Jonatha next asks the students, “What would happen if I changed the order of the numbers I added?” while she moves the cubes. The students chant 15. She does this a few times then asks the students to explain why the answer does not change. Tim responds, “Because you’re not taking anything away or adding anything.” Jonatha restates his answer, then while moving the cubes she asks, “Sean how many cubes do I have now?” Sean answers, “15.”

“Correct, because I have not added or taken any cubes away.” Jonatha rephrases the students answer. Wilhelmina adds, “Because it’s the same sum.” Emphasizing the word sum, which Jonatha had not used.

Jonatha transitions to the white board. She then tells the students that she will show Ralph’s thinking on the boards. The class solves the problem again while Jonatha writes the total of four plus three then eight plus seven using the doubles-plus-one strategy. Once again, the class reviews the procedure for adding three numbers. Jonatha points out that 4 and 3 are doubles-plus-one. She and the students sing a song describing how to figure out when numbers are doubles plus one. When the problem has been completed Jonatha asks the students to assess if Wilton’s thinking had been correct.
Next Jonatha tells the students that they will be practicing adding three numbers and asks the class to return to their seats, while she passes out material. Jonatha hands out tokens and work mats to students and asks them to count their counters.

“I’m going to give you three numbers and I want you to place that number in the box on your mat.” She then asks the students to write the number in boxes on their work mat. Students are to slide the top two numbers and slide those two numbers into another box. Jonatha demonstrates with magnetic counters on the board. As students try to slide and add numbers on their mats, mass confusion ensues, even though many students have figured out the final answer. They seem confused by the procedure of taking two numbers then taking that sum and adding that to the final number.

Jonatha stops the students. She tells them that she has made a mistake in explaining what they are to do and asks them to return to the carpet to provide further group practice. When the students are settled, she asks the students, “Can we add numbers in any order?” The students chorus, “YES!” On the board, she works three more problems asking students to use addition strategies of counting on, doubles, and doubles plus one to figure out the sums.

It seems that most of the students understand that addition means grouping numbers to get the total. But they are stuck when it comes to using the strategy of adding two numbers by sliding them to a box on the side then adding the final addend with the number in the box. The class ends because the students go to physical education.

In comparing the description and the use of excellent strategies, I saw Jonatha carefully attending to students’ needs (Working with Students and Mathematics). She, she noticed student errors, when the students made a mistake during independent work,
she quickly called them back to the carpet and addressed their misconceptions. In most segments Jonatha is adept at using the Richness of the Mathematics strategies. She uses Multiple Representations, when the students add three-digit numbers using cubes, drawing, and mental math. She also uses Explanations in most segments when, she shows the students examples with cubes that they can add (3+4) + 8 or 4 + (3+8). She then asks the student why the answer stays the same. They reply, “Because we didn’t add or take anything away.” In general, the chart shows that Jonatha used the Richness of the Mathematics strategies better than Working with Students and Mathematics or Common Core-Aligned Student Practices. A detailed summary analysis for Observation One can be seen in Table 6.

**Observation Two - Grade One**

Jonatha is teaching a new group of first-graders. The students have lots of energy and need a lot of redirection. She keeps the class moving quickly with short reminders that guide student behavior and attention. This is a measurement lesson where the students are learning to use the words longest and shortest to compare object lengths. The first five minutes of the lesson the students play the “Around the World” math game to practice addition and subtraction facts. The game is played for about five minutes of the class period. Although the students seem to know how to play this game, before the game begins Jonatha reminds the students that this game will contain addition and subtraction facts between 10 and 20.

After the game, students come to the carpet for math instruction. Jonatha tells the students that they are starting a new unit with this lesson. “We are starting a brand-new unit today. We started this unit last Thursday with a story.”
Table 6

*MQI Analysis for Observation One – Grade One*

<table>
<thead>
<tr>
<th>MQI Dimensions</th>
<th>Segments</th>
<th>Scores</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>RICHNESS OF THE MATHEMATICS</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Linking Between Representations</td>
<td>S1</td>
<td>N</td>
<td>The beginning of the class time was used to review math facts. The rest of the class period the teacher and students used counters and digits to represent addition concepts.</td>
</tr>
<tr>
<td></td>
<td>S2 – S4</td>
<td>H</td>
<td></td>
</tr>
<tr>
<td>Explanations</td>
<td>S1</td>
<td>N</td>
<td>The teacher provided explanations of concepts, associative and commutative properties of addition. She used different colored cubes to represent numbers, and then asked students what happens when you change the number order.</td>
</tr>
<tr>
<td></td>
<td>S2 – S4</td>
<td>H</td>
<td></td>
</tr>
<tr>
<td>Mathematical Sense-Making</td>
<td>S1</td>
<td>N</td>
<td>Teacher provided meaning to procedures used in adding three addends. She described why a doubles-plus-1 strategy is useful in solving addition problems.</td>
</tr>
<tr>
<td></td>
<td>S2</td>
<td>H</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S3</td>
<td>N</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S4</td>
<td>H</td>
<td></td>
</tr>
<tr>
<td>Multiple Procedures or Solutions Methods</td>
<td>S1</td>
<td>N</td>
<td>Students provide multiple methods for solving addition of three addends. She provided explicit description and discussion for adding three single digit-numerals including adding the numbers in order.</td>
</tr>
<tr>
<td></td>
<td>S2 – S4</td>
<td>M</td>
<td></td>
</tr>
<tr>
<td>Patterns and Generalizations</td>
<td>S1</td>
<td>M</td>
<td>Teacher provides students with addition solution strategies; count on, doubles, and doubles plus 1.</td>
</tr>
<tr>
<td></td>
<td>S1</td>
<td>H</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S1</td>
<td>N</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S1</td>
<td>M</td>
<td></td>
</tr>
<tr>
<td>Mathematical Language</td>
<td>S1</td>
<td>M</td>
<td>Teacher and students use appropriate mathematical language throughout lesson including using the word sum, and addends.</td>
</tr>
<tr>
<td></td>
<td>S2</td>
<td>H</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S3</td>
<td>H</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S4</td>
<td>M</td>
<td></td>
</tr>
<tr>
<td>WORKING WITH STUDENTS AND MATHEMATICS</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Remediation of Student Errors and</td>
<td>S1</td>
<td>L</td>
<td>Teacher and students use appropriate mathematical language throughout lesson including using the word sum, and addends.</td>
</tr>
<tr>
<td>Difficulties</td>
<td>S2</td>
<td>L</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S3</td>
<td>M</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S4</td>
<td>H</td>
<td></td>
</tr>
<tr>
<td>MQI Dimensions</td>
<td>Segments</td>
<td>Scores</td>
<td>Notes</td>
</tr>
<tr>
<td>--------------------------------</td>
<td>----------</td>
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<td>---------------------------------------------------------------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>Teacher Uses Student</td>
<td>S1</td>
<td>N</td>
<td>Teacher allowed students to express some mathematical concepts and used their ideas to promote understanding using questions.</td>
</tr>
<tr>
<td>Mathematical Contributions</td>
<td>S2</td>
<td>H</td>
<td>She asks the students why nothing changed whether she added 7 + 8 or 8 + 7. Tim replies that they didn’t add or take anything away.</td>
</tr>
<tr>
<td>S3</td>
<td>M</td>
<td></td>
<td></td>
</tr>
<tr>
<td>S4</td>
<td>M</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**COMMON CORE-ALIGNED STUDENT PRACTICES**

| Student Provide Explanations   | S1       | N      | The students explain why sum did not change even if the order of addends changed in segment S2. Students did not provide explanations in the other segments. |
| S2                             | H        |        |                                                                                                                                         |
| S3                             | N        |        |                                                                                                                                         |
| S4                             | N        |        |                                                                                                                                         |

| Student \Mathematical Questioning and Reasoning | S1       | L      | Students provided mathematical reasoning when given a rational why sums do not change when the order of addends changes.                 |
| S2                             | H        |        |                                                                                                                                         |
| S3                             | N        |        |                                                                                                                                         |
| S4                             | N        |        |                                                                                                                                         |

| Student Communicate about the Mathematics of the Segment | S1       | L      | Although students answered several questions during each of the segments, most student communication was Inquiry-Response-Evaluation (IRE) type. Only in the second segment did students provide an explanation in their communication. |
| S2                             | M        |        |                                                                                                                                         |
| S3                             | L        |        |                                                                                                                                         |
| S4                             | L        |        |                                                                                                                                         |

| Task Cognitive Demand          | S1       | N      | Student tasks were either routine or heavily scaffolded with directions. Teacher did most “heavy lifting” by asking leading or closed questions. |
| S2                             | L        |        |                                                                                                                                         |
| S3                             | N        |        |                                                                                                                                         |
| S4                             | N        |        |                                                                                                                                         |

| Student Work with Contextualized Problems | S1       | N      | The second segment contained a contextualized problem and students play a role in deciding how to solve the problem.                   |
| S2                             | L        |        |                                                                                                                                         |
| S3                             | N        |        |                                                                                                                                         |
| S4                             | N        |        |                                                                                                                                         |
With the students’ help, Jonatha retells the story of a king wanting to build a bed for his wife and that the bed ended too small because the king and the carpenter’s feet were different sizes. She asks the students to recall that when you measure something your units must be the same size. Jonatha shows students the words longest and shortest.

Next, Jonatha reads a book about snakes to students. The book explains what inches, feet, yards, and miles mean. The book continues to describe different kinds of snakes by their size. The book repeatedly uses the word longest to compare different kinds of snakes. Jonatha stops reading the book and tells students they will be measuring snakes and comparing which snakes are the longest and which are the shortest.

Then she uses lines on the board for students to understand that long and short describe a size, while longest and shortest are used to compare more than one thing. Using snake length models made of strings, Jonatha and the students compare snake lengths by using the words shortest and longest. After laying out three snakes she asks students to compare which snake is the shortest and which is the longest. Each time the students chorus the answer. She repeats the procedure several times. This activity engages the students and they all seem to be able understand the meaning and how to use the words “longest’ and shortest’.

Jonatha has run out of time. The students return to their seats. She explains into the camera that when they return from physical education the students will label and draw three lines and label two of them as shortest or longest.

To teach the concept of longest and shortest, Jonatha has activated the students’ interest and emotion by using snakes and their size. Students can better understand a new concept when it is connected to prior knowledge, interests them, or is emotionally
charged (Zirbel, 2006). By using snakes, Jonatha has activated students’ prior knowledge; there are snakes in Florida, she has chosen something that the students are excited to learn about, and because the topic is snakes, it seems to instinctually arouse the students’ emotional level.

An observational analysis shows that in this measurement lesson Jonatha has relatively high usage of Mathematical Language. She uses comparative words – e.g. long, longer, longest, and short, shorter, shortest to compare the length of the snakes in the book. The observational analysis shows that the other instructional strategies were not used at a high level of quality. Most strategy usage was coded at mid or low quality. For example, all segments contained student communication, but the student communication consisted of short answers without students providing explanations; they chorused mathematics facts or answers like longest or shortest. A detailed summary analysis for Observation Two can be seen in Table 7.

**Observation Three - Grade One**

Iris teaches a first- and second-grade combination class with sixteen students. The school has a full first-grade, a full second-grade, and a combination class. The students in the combination class were chosen randomly with a balance of 8 first-graders and 8 second-graders. Iris’s classroom is cheerful and well organized. She has a loving and gentle manner with her students. Today, because she has a student teacher, she will be teaching only the first-graders.
Table 7

*MQI Analysis for Observation Two – Grade One*

<table>
<thead>
<tr>
<th>MQI Dimensions</th>
<th>Segments</th>
<th>Scores</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>RICHNESS OF THE MATHEMATICS</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Linking Between Representations</td>
<td>S1</td>
<td>N</td>
<td>Segment 1 consisted of math fact practice game. Segment 2 linked measurement with using a person’s feet and a unit of measurement of feet.</td>
</tr>
<tr>
<td></td>
<td>S2</td>
<td>L</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S3</td>
<td>N</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S4</td>
<td>N</td>
<td></td>
</tr>
<tr>
<td>Explanations</td>
<td>S1</td>
<td>N</td>
<td>Teacher provides explanation of measurement words. Rose emphasizes that measurement units must be the same size.</td>
</tr>
<tr>
<td></td>
<td>S2</td>
<td>N</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S3</td>
<td>M</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S4</td>
<td>L</td>
<td></td>
</tr>
<tr>
<td>Mathematical Sense-Making</td>
<td>S1</td>
<td>N</td>
<td>The teacher gives examples comparing lengths she calls the words longest and shortest as measuring words. She explains that short shows a size and shortest shows a comparison.</td>
</tr>
<tr>
<td></td>
<td>S2</td>
<td>N</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S3</td>
<td>L</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S4</td>
<td>L</td>
<td></td>
</tr>
<tr>
<td>Multiple Procedures or Solutions Methods</td>
<td>S1 – S4</td>
<td>N</td>
<td>Segments did not contain multiple the uses of procedures or solution methods.</td>
</tr>
<tr>
<td>Patterns and Generalizations</td>
<td>S1 – S4</td>
<td>N</td>
<td>Although some patterns were mentioned, these patterns were not developed during any of the segments.</td>
</tr>
<tr>
<td>Mathematical Language</td>
<td>S1</td>
<td>N</td>
<td>This was primarily a vocabulary lesson. Students reviewed the use of standardized measurement and the terms longest and shortest.</td>
</tr>
<tr>
<td></td>
<td>S2 – S4</td>
<td>H</td>
<td></td>
</tr>
<tr>
<td><strong>WORKING WITH STUDENTS AND MATHEMATICS</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Remediation of Student Errors and Difficulties</td>
<td>S1 – S4</td>
<td>N</td>
<td>The students did not encounter any difficulties</td>
</tr>
</tbody>
</table>
| Teacher Uses Student Mathematical Contributions | S1 – S4 | N      | The students did not make meaningful contributions to push the understanding forward. Teacher questions required only one answer. Students were only required to use terminology or remember teacher given information.
<table>
<thead>
<tr>
<th>MQI Dimensions</th>
<th>Segments</th>
<th>Scores</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Student Provide Explanations</td>
<td>S1 – S4</td>
<td>N</td>
<td>The students did not provide explanations in these segments.</td>
</tr>
<tr>
<td>Student Mathematical Questioning and Reasoning</td>
<td>S1</td>
<td>N</td>
<td>Students in these segments were not required to make claims, justify why they disagree, or form conclusions based on patterns.</td>
</tr>
<tr>
<td></td>
<td>S2</td>
<td>L</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S3</td>
<td>N</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S4</td>
<td>L</td>
<td></td>
</tr>
<tr>
<td>Student Communicate about the Mathematics of the Segment</td>
<td>S1 – S4</td>
<td>L</td>
<td>The segments primarily featured teacher explanations with minimal student one-or-two words occur regularly during these segments.</td>
</tr>
<tr>
<td>Task Cognitive Demand</td>
<td>S1 – S4</td>
<td>N</td>
<td>Students were engaged in cognitively undemanding tasks, such as recalling math facts, listening to teacher presentation</td>
</tr>
<tr>
<td>Student Work with Contextualized Problems</td>
<td>S1 – S4</td>
<td>N</td>
<td>The students did not work on contextualized problems.</td>
</tr>
<tr>
<td>Student Provide Explanations</td>
<td>S1 – S4</td>
<td>N</td>
<td>The students did not provide explanations in these segments.</td>
</tr>
<tr>
<td>Student Mathematical Questioning and Reasoning</td>
<td>S1</td>
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<tr>
<td></td>
<td>S2</td>
<td>L</td>
<td></td>
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<tr>
<td></td>
<td>S3</td>
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</tr>
<tr>
<td></td>
<td>S4</td>
<td>L</td>
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<td>Student Communicate about the Mathematics of the Segment</td>
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<td>S1 – S4</td>
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<td>The students did not work on contextualized problems.</td>
</tr>
</tbody>
</table>
Instruction Using Word Problems

Iris’s math class started with a group word problem. The students opened their math journal and paste the given word problem on to the top of the first right hand page in their journal. The class reads the problem together. Then they reread the problem looking for information the problem gives them, and the question the problem is asking. She makes sure that students can read the words correctly. They chorally read the word problem. The problem was: “Katie had a picnic for her friends. She bought 50 hamburgers and 20 hotdogs. How many hotdogs and hamburgers did she buy in all?”

After reading the problem she then reminds students that she doesn’t want the answer; she admonishes them that they are going to think through it. Iris calls on students to give her the information, which she writes on the board. As the students are giving her the information she writes it on the board. One student tells her that she bought 50 hamburgers; another student tells her that she had 20 hotdogs. Iris then asks for the question, and a student reads it, “How many hamburgers and hotdogs in all? Who can tell me what that question means?” she asks the class. A student answers add, and she follows that up with, “How do you know we are going to add.” Student One says, “We have to add because 50 is bigger than 20.”

Iris follows that up with anybody else having any other ideas. She does not address the student’s error. The next student’s answer is, “Because it says, ‘in all.’” Iris gives him a big smile, letting everyone know that was the correct answer.

Moving her hand in a circular motion in front of her, she asks the students, “What does ‘in all’ mean we have to do?” She continues prompting students, “That means we have to put it all…” Students chorus, “All together.”
Next, she asks, “Students what would be a quick picture that I can draw to help me see how I can do this? Esther?” Iris rephrases Esther’s answer, “Okay, you could draw lines and circles. Then how would I show 20 hotdogs?” Esther continues, “You could draw two lines.” Iris asks her if she would do anything else. Emma answers, “Because each line has 10 in it.”

Iris moves on to other students, “What do I do next? Amelia.” Amelia answers, “You need to do five lines.” Iris presses her for a more complete answer. “You need to draw five lines for what? Amelia answers, “For 50.”

Iris spends some time discussing where to place the next five lines. The lines are placed in the same row as the two lines with a gap between them as if the two lines and the five lines were separate words in the same sentence.

Again, she asks the class, “Why don’t I have any dots on this?” Lui Na answers, “Because there are no ones, twos or threes.”

Next Iris asks the student, “How will I write the number sentence?” Quickly, a student provides an answer, “70.” Iris replies, “Okay, good job you counted on by…?” The student finishes her sentence, “tens”.

“I know a different way,” a student calls out. “two plus five equals seven, then you just add a zero.” Iris smiles, nods, and restates what Sidney said, while pointing to the numbers written on the board. The next student starts her explanation, “Because a five—uh-mm—because a number plus two equals two more.” Iris encourages her to use the numbers on the board. Amelia continues, “Like you have to count 20 then you have to count 50 and then you have to, count 60 then 70. Like two then five, and then you would
have to count just two more numbers so 50 would equal 60 then 70. Iris nods and give the explanation a name, “Okay, you counted on then.”

Iris asks the class for another way of finding the answer by looking at the quick picture. Esther answers very shyly and softly, but Iris restates, “Esther says that we can use the line to count by tens.” She has the students count as she points to the lines. Then she reminds the students that they have been learning many strategies that they can use for addition.

Next Iris works with the class in composing a sentence to answer the question. After writing the sentence she provides students with sentence stems and gives them time to write an explanation of how they came up with their answer. Iris walks around as students write their explanations, and helps students complete their sentences.

Small Group Instruction - Group One

Iris calls the first group to her table. The rest of the students are working on math independently. Their first task is to review math vocabulary. Students provide the definition of addend, sum, and part, part, whole. Next, she asks the student to use base-10 blocks to make the number 26. Once the students have completed the task she asks them why they used 2 tens rods. The student replies that 26 has 2 tens and the value of each rod is 10.

Then she asks them to make 26 another way using the base-10 blocks. When she asks the students what they did, they explain that they took away one of the tens. Iris makes 26 using the base-10 blocks and takes 10 away. She asks the students, “How many blocks do I have if I take it away?” The students respond “16”. She replaces the tens
block and asks the students again, “Did we take it away?” This time Dennis says, “We made a 10 with the ones.” “Right, right, I didn’t just take it away, I broke it apart,”

“How can I say this (pointing to the base take blocks) using the words tens and ones,” she asks the students next. “Patrick says, 2 tens and 7 ones.” Iris slowly separates the ones cubes and has him count them. Patrick corrects himself. Next, she asks Gabrielle how to say it using the words tens and ones. She says 1 ten and 16 ones. Iris reminds her that is what they had before when they separated the tens, but now it is 2 tens and 6 ones.

Next Iris asks the students to use two colors and make seven and one. Then the students are asked what is seven plus one. All the students reply “eight”. Now make them 70 and 10, while keeping seven and one. “Now I want you to add the 7 ones and 1 ones. How much is that?” The students answer “eight”. Now add the 7 tens and 1 ten. How much is that? The students answer “80.” Now the students compare the 7 ones and 1 ones and 7 tens. Iris shows the students how to write a number sentence, 70 + 10 = 80. She shows the students how they will use quick pictures, writing tens and ones, to answer the word problems found in the math textbook. They do the first problem. She asks them how they figured out the problem.

Then she reads the next problem and they work it together. Finally, they read the higher order-thinking question and review the thinking they had to do to figure it out. Then they go to their desks to work.

Small Group Instruction - Group Two

After the first group leaves and the second group settles in, the lesson begins.

Teacher: “What do we call these?”

Students in union: “Ones.”
Teacher: “How many of these (points to some ones in her hand) make these (pointing to 1 tens in her other hand), Tina?”

Several other students: “10.”

Teacher: “Tina?”

Tina: “Ten.”

Teacher: (Placing three base-10 blocks on the table.) “If I’m counting by tens how many do I have?”

Several Students: “30.”

Then the students count by tens as the teacher points to the base-10 blocks.

Quickly, Iris changes the activity; she tells the students that they will be working on addition today and reviews the math vocabulary of addends and sum from the previous day. Next the students are asked to show her the number six using the base-10 bars or the unit cubes.

Teacher: “What did you use to make six?”

Students: “Ones.”

Teacher: “Count them.”

Students in unison: “one, two, three, four, five, six.”

Teacher: “We have six?”

Students: “Ones.”

Teacher: “Now I want you to show me 60. How are you going to show me 60?”

Teacher: (After students have completed the task.) “I noticed you all used theses (pointing to the tens) to make 60. Why did you use these (pointing to the tens) to make 60?”
Student: “Because it’s the tens.”

Teacher: “Why did you use these to make 60, Esther?”

Esther: “Because it’s easier than getting 60 ones.”

Teacher: (Restates what Esther said.) “Because it’s easier,” (pointing to the ones blocks) “she would need 60 of these. Tina why did you use these” (pointing to the base-10 blocks)?”

Tina does not give an answer. Iris now asks her to tell her another way of writing 60 and writes the number 60 on a small board. After she writes the number; she asks for another representation of 60. Tina is still silent. Esther answers, “Draw six sticks.” She asks for another way to say 60 using the word tens. Nate answers “6 tens.” Iris points out that there are different ways of writing 60, “you can use the digits, you can draw six sticks representing ten, and you can write 6 tens.”

This group needs more practice with different representations of tens. Iris has them repeat the procedure with the number 35. As they tell her the ways they can represent 35 she keeps drawing them back to their representation of 60 to compare the representations of 60 with 35. Finally, Tina can explain how many tens and ones in each numeral.

After this review, Iris has the students begin the lesson. In the lesson, students are to generalize and notice the pattern between $7 + 1 = 8$ and $70 + 10 = 80$. This group reacts quite differently than the other group. They genuinely seem amazed by the correlation between the equations, but they can’t seem to articulate what the pattern is until Iris writes the equation on the board.
Teacher: “What’s the difference between this one (pointing to 1) and this one (pointing to 10)? And the difference between this seven (pointing to 7) and this seven (pointing to 70)? And the difference between this eight (pointing to 8) and this eight (pointing to 80)?”

Student A: “These ones have a zero at the end.”

Teacher: “But what’s the difference?”

Student: B (Using a questioning tone.) “It’s more.”

Teacher: “But how do you know it’s more?”

Student B: “Because it’s seventy.”

Teacher: “It’s seventy, but what do you call this seven?” (pointing to the seven in 70.)

After Iris repeats the question three more times pointing to the numbers, the students answer, “It’s 7 tens.” One student cheers, “Yeah.” Iris continues to point to the different numbers on the board to have the students identify what each number means and the relationship between the different representations of the numbers. After some more practice another student says, “I got it!” Now they all seem to understand. Their answers are confident now. The students seem to understand that they now understand something they didn’t know before and are proud of themselves.

At this point, Iris brings the students back to the goal of the lesson. Pointing to the equations on the board she asks, “What helped you know that 70 plus 10 equals 80?” After a student explains their reasoning Iris then rephrases what she said, “Because if you knew that 7 plus 1 equals 8 then you could figure that 7 tens plus 1 tens equal 8 tens.” Iris gives students time to think and explain their thinking, but is also careful to rephrase their
sometimes-jumbled words into clear sentences, as a way of checking for understanding. These students rehearse the concept several more times. After each problem, she asks a different student to explain their thinking. Now the students are ready to use their textbooks and do the problems in the book with confidence. As the students work in their book one student calls out, “This is so easy!”

Small Group Instruction - One-to-One

While the next group is coming, Iris reminds the rest of the class what they are supposed to be doing and quickly turns her attention to Tina. At the beginning of her group’s lesson Tina had struggled. Now Iris provides her with one-on-one instruction. Iris asks Tina to do the same thing but in a different way, given a four and a one what changes can we make to get 50. Tina is silent; Iris waits for a few seconds, and then writes on the board 4 + 1 = 5. She then gives Tina a clue. “I want to use tens.” In a tentative and questioning voice Tina replies, “40,” and she pauses, and Iris re-voices her answer, “So 40 plus”, as she writes 40 + on the board and then waits for Tina to complete the equation. In the same hesitant tone, she says, “10,” and pauses again. Iris writes the number, pauses and looks toward Tina. As she waits for Tina to continue, Iris provides her with more scaffolding, as she says and writes “=.” Finally, Tina with a little more confidence says “50.”

Iris doesn’t stop there but continues to probe, “What did I use instead of ones?” When Tina answers tens. Iris confirms her answer, “Yes,” without stopping she picks up the base-10 block to show as she asks, “One 10 and 4 tens makes what? With more confidence Tina answers, “50”. Iris keeps pressing, “It makes 50 or 5?” Tina once again
tentatively answers, “Tens.” Now Iris directs Tina to her textbook, “What are you going to write now?” she says pointing to the problem on the page.

With everyone in the next group at the table, Iris quickly checks Tina’s work, asks her if she understands, and sends her to work on the rest of the problems on her own.

Small Group Instruction - Group Three

Group Three begins with quick directions to the students doing independent work to stay on task. Then her attention quickly turns back to the math group. “We are going to work with tens and ones today. Show me the number 35.” The students use base-ten blocks to make 35.

Once the students have completed the task, she asks, “What did you do to make 35? Can you describe it to me?”

Student: We all did 35, but we all did it a different way.

Teacher: “Darlene, how did you it?”

Darlene: “I made it with 2 tens and 15 ones.”

Student: “Three.”

Teacher: “How many ones did you use?”

Student: “Five.”

Now Iris asks the students to make 7 ones in one color and 1 one in another color. As they are building their numbers, one student says, “Okay, well that will be eight.” Iris now asks, how will I write that in a number sentence. “That will be seven plus one equals eight,” quickly answers one of the students. Iris continues, “We talked about this yesterday. Which is my sum?” “The eight” answers a student. “Which are addends?” The students answer in chorus, “The seven and the one.” Iris continues, “Seven and one are
addends and eight is the sum. Now she asks the same question in a different way, “The sum of seven and one is?” A student answers, “Eight.”

Teacher: Now I want you to keep the seven and one on the table and above put the 7 tens and 1 tens.”

This group is one step ahead. One student says, “I know what it equals.” Another student calls out, “80.” This creative group needs very explicit instructions, “I want 7 tens above your 7 ones.” She continues, “I want to know what the difference is if I have 7 ones and 1 one and we said it makes eight. Now I have 7 tens. What does 7 tens make?” A student answers, “Seven tens make 70,” as Iris writes the number on the board.

Teacher: “What does 1 ten make?”

Student: “10.”

Teacher: “But when I put these together what do I get?”

Student “80.”

Teacher: “Look at the board here. What do you notice about these two number sentences?”

One student gasps, another student waves her hand up to be called on, and one student blurts out, “I know! I know!”

Although the students see that there is the pattern, they have trouble putting it into words. Amelia says, “That’s the seven and the 70,” while pointing at the board, but then she stops. Iris asks, “Is that the same?” Amelia repeats, “That’s 7 + 1 = 8 and 70 + 10 = 80.” Iris accepts that answer, but allows another student to add on to what Amelia has said. She asks, “What do you notice that is the same about them?” Chrystal answers, “They all have zeros at the end.” Iris leads the students to pay attention to the pertinent
qualities, while circling the number sentences. She says, “Look at this number sentence here, and look at this number sentence here. What do you see that’s the same in this sentence and this sentence?”

Student 1: “There’s a one and a one that’s the same.”

Student 2: “The first one has a one and the second one has a 10.”

Moving the students on to notice the important element between the two sentences, Iris asks, “How can we write this (70 + 10 = 80) using the word tens?”

Eric: “Seven tens and 1 tens equals 8 tens.”

Iris now reviews with the students. She reminds them that they figured out that seven plus one equals and that the student said the number sentence had helped them figure out 70 + 10 = 80. And how it was possible to write that number sentence in another way, 7 tens + 1 ten = 8 tens. She continues by checking for understanding and asking students how many tens in 90, in 50, in 40. The last student asked, “Say four instead of 4 tens?” Iris makes a point of having him say it correctly, “Not four but 4 tens.”

Now the students are directed to open their textbooks. She tells them that in today’s lesson they will be adding tens. The students have had sufficient practice that when they read the word problem with tens they are able to draw tens, write a number sentence, and answer how many tens.

When the students attempt to complete the problems on their own, Iris notices that two students have put 30 + 40 = 70 tens. She grabs two fistfuls of base-ten rods as a visual. Then she says, “That means I would need 70 rods. Is that what we have? We have 70 tens? Let’s look at it again. We did 30 + 40. How many tens is that?” One student calls out, “70” Iris repeats, “Is it 70 tens or is it 7 tens?”
Iris must interrupt her lesson; Eric is not paying attention and the classroom is getting noisier. She reviews with the students: “We added \(30 + 40 = 70\).” Now if we want to say it in tens when must we write it using the word tens we have how many tens are there? After the students understand that 70 equals 7 tens, Iris ends the lesson.

As noted in Table 8, students try to make sense of why 4 tens + 1 ten = 5 tens or 50 ones. Often Iris rephrases student comments to clarify the students’ ideas. As she does when she says, “Because if you knew that seven plus one equals eight then you could figure out that 7 tens plus 1 tens equals 8 tens. In this and other instances Iris helps students generalize about the concept they are learning. Throughout the lesson, Iris uses mathematical language, e.g. ones, tens, digits, sum, part-part-whole. And she encourages students to use mathematical language as they explain their thinking.

In Working with Student and Mathematics Iris consistently uses student contributions. After using a pictorial representation to add 50 + 20, Sidney calls out, “I know a different way, you can add two plus five equals seven then add a zero to the end. Iris accepts this contribution, asks the student why this is. Another student continues the explanations, ‘Like you have to count 20 then you have to count 50 and then you have to count 60 then 70.’”

From this, one can see that Iris is very competent in the use of the Richness of the Mathematics dimension, but she is not as strong in her usage of the Common Core-Aligned Student Practices. A detailed summary analysis for Observation Three can be seen in Table 8.
Table 8

MQI Analysis for Observation Three – Grade One

<table>
<thead>
<tr>
<th>MQI Dimensions</th>
<th>Segments</th>
<th>Scores</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>RICHNESS OF THE MATHEMATICS</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Linking Between Representations</td>
<td>S1</td>
<td>N</td>
<td>Segment 1 consisted of math fact practice game. Segment 2 linked measurement with using a person’s feet and a unit of measurement of feet.</td>
</tr>
<tr>
<td></td>
<td>S2</td>
<td>L</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S3</td>
<td>N</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S4</td>
<td>N</td>
<td></td>
</tr>
<tr>
<td>Explanations</td>
<td>S1</td>
<td>N</td>
<td>Teacher provides explanation of measurement words. Rose emphasizes that measurement units must be the same size.</td>
</tr>
<tr>
<td></td>
<td>S2</td>
<td>N</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S3</td>
<td>M</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S4</td>
<td>L</td>
<td></td>
</tr>
<tr>
<td>Mathematical Sense-Making</td>
<td>S1</td>
<td>N</td>
<td>The teacher gives examples comparing lengths she calls the words longest and shortest as measuring words. She explains that short shows a size and shortest shows a comparison.</td>
</tr>
<tr>
<td></td>
<td>S2</td>
<td>N</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S3</td>
<td>L</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S4</td>
<td>L</td>
<td></td>
</tr>
<tr>
<td>Multiple Procedures or Solutions Methods</td>
<td>S1 – S4</td>
<td>N</td>
<td>Segments did not contain multiple the uses of procedures or solution methods.</td>
</tr>
<tr>
<td>Patterns and Generalizations</td>
<td>S1 – S4</td>
<td>N</td>
<td>Although some patterns were mentioned these patterns were not developed during any of the segments.</td>
</tr>
<tr>
<td>Mathematical Language</td>
<td>S1</td>
<td>N</td>
<td>This was primarily a vocabulary lesson. Students reviewed the use of standardized measurement and the terms longest and shortest.</td>
</tr>
<tr>
<td></td>
<td>S2 – S4</td>
<td>H</td>
<td></td>
</tr>
<tr>
<td><strong>WORKING WITH STUDENTS AND MATHEMATICS</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Remediation of Student Errors and Difficulties</td>
<td>S1 – S4</td>
<td>N</td>
<td>The students did not encounter any difficulties</td>
</tr>
<tr>
<td>Teacher Uses Student Mathematical Contributions</td>
<td>S1 – S4</td>
<td>N</td>
<td>The students did not make meaningful contributions to push the understanding forward. Teacher questions required only one answer. Students were only required to use terminology or remember teacher given information.</td>
</tr>
<tr>
<td>Student Provide Explanations</td>
<td>S1 – S4</td>
<td>N</td>
<td>The students did not provide explanations in these segments.</td>
</tr>
<tr>
<td>MQI Dimensions</td>
<td>Segments</td>
<td>Scores</td>
<td>Notes</td>
</tr>
<tr>
<td>----------------------------------------------------</td>
<td>----------</td>
<td>--------</td>
<td>-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>Student Mathematical Questioning and Reasoning</td>
<td>S1 – S4</td>
<td>L</td>
<td>The segments primarily featured teacher explanations with minimal student one – or – two words occur regularly during these segments.</td>
</tr>
<tr>
<td>Student Communicate about the Mathematics of the Segment</td>
<td>S1 – S4</td>
<td>L</td>
<td>Students were engaged in cognitively undemanding tasks, such as: recalling math facts, listening to teacher presentation</td>
</tr>
<tr>
<td>Task Cognitive Demand</td>
<td>S1 – S4</td>
<td>N</td>
<td>The students did not work on contextualized problems.</td>
</tr>
<tr>
<td>Student Provide Explanations</td>
<td>S1 – S4</td>
<td>N</td>
<td>The students did not provide explanations in these segments.</td>
</tr>
<tr>
<td>Student Mathematical Questioning and Reasoning</td>
<td>S1 – S4</td>
<td>N</td>
<td>Students in these segments were not required to make claims, justify why they disagree, or form conclusions based on patterns.</td>
</tr>
<tr>
<td>Student Communicate about the Mathematics of the Segment</td>
<td>S1 – S4</td>
<td>L</td>
<td>The segments primarily featured teacher explanations with minimal student one – or – two words occur regularly during these segments.</td>
</tr>
<tr>
<td>Task Cognitive Demand</td>
<td>S1 – S4</td>
<td>N</td>
<td>Students were engaged in cognitively undemanding tasks, such as: recalling math facts, listening to teacher presentation</td>
</tr>
<tr>
<td>Student Provide Explanations</td>
<td>S1 – S4</td>
<td>N</td>
<td>The students did not provide explanations in these segments.</td>
</tr>
<tr>
<td>Student Mathematical Questioning and Reasoning</td>
<td>S1 – S4</td>
<td>N</td>
<td>Students in these segments were not required to make claims, justify why they disagree, or form conclusions based on patterns.</td>
</tr>
<tr>
<td>Student Communicate about the Mathematics of the Segment</td>
<td>S1 – S4</td>
<td>L</td>
<td>The segments primarily featured teacher explanations with minimal student one – or – two words occur regularly during these segments.</td>
</tr>
<tr>
<td>Task Cognitive Demand</td>
<td>S1 – S4</td>
<td>N</td>
<td>Students were engaged in cognitively undemanding tasks, such as: recalling math facts, listening to teacher presentation</td>
</tr>
<tr>
<td>Student Provide Explanations</td>
<td>S1 – S4</td>
<td>N</td>
<td>The students did not provide explanations in these segments.</td>
</tr>
</tbody>
</table>
Observation Four - Grade One

The math class starts with students gathered at the carpet while Marta calls on individual students to recall the numbers that make 10 from a 10-frame. Students are shown two dots and they respond, “Eight plus two,” making an expression that equals ten. She shows each 10-frame for a few seconds then calls on a student to provide the correct expression.

Marta asks the students, “What are we making?”

“Ten,” the class responds.

Then she asks, “What are we finding?” She answers the question, “The missing addend.”

Marta uses this activity to link today’s lesson with previous learning. She reminds the students that the last unit they studied strategies for adding numbers to 10. She tells them that now they will be starting a new chapter where they will be studying how numbers work, number sense, the value of numbers.

Marta gives each student a hundreds chart and asks them to count and touch the numbers to twenty. After the students count to 20 she asks them, “I want you to look closely at the numbers we counted. What did you notice about those numbers?”

Jane: “The numbers on the top [row] they don’t have one. And these numbers [pointing to the second row] they all have one.”

Marta rephrases what Jane said being somewhat more precise and pointing out that the digits one through nine are numbers by themselves, but the individual digits in 10 to 20 are not numbers by themselves. Now she asks the students, “How many numbers are there?”
Now she asks the students to touch the one. She tells the students that it is called a
digit. Next, she asks the class to identify how many digits are in the number three. One-
student answers one. She reinforces that answer by saying that three is a single digit.
Marta links back to what Jane said and reminds the students that when we get to number
ten things change. “How many digits are there [in the number ten]?” Marta asks. After a
student response of two, she asks, “What are the digits that you see?”
Pete answers, “One and zero.”

Marta asks the students to put their finger on the number 14. She asks Anthony to
describe what he sees, and then quickly asks him how many digits are there in the
number 14. Anthony says nothing so she asks him again. “Do you see two numbers or
one number?”

Anthony says, “Two numbers.”

“That’s two numbers or two digits,” Marta moves on and asks Sidney, “What are
the two numbers that you see in the number 14.”

Sidney says, “A one and a four.”

Marta tells the class that today’s lesson involves learning what the one means and
what the four means in 14. “We’re going study the place value”. She asks the students to
return to their seats, get their markers and white board, and return to the carpet when they
have their material.

Once students are settled, Marta tells the students that they will explore one of the
numbers they just counted. She asks the students to make a box in the lower right-hand
corner and write the number 14 in it. Then they make 14 circles in the rest of the board.
She asks Sidney, “How many circles did you make?”
Sidney says, “Fourteen.”

Marta points to the four as she asks the student to circle that many. She repeats the directions and watches as the students complete their task. She gives students a thumbs-up when they have circled four. She asks one student, “Parker, please circle four.” She continues to give students a thumbs-up. Once she has checked the work of the rest of the students, she gets up and goes to Parker. She helps Parker count the four circles and then circle them.

“Okay, that number that you circled showed how many were on this side of the number, [Pointing to the four.] Would you circle this number? [Pointing to the one in the tens place.]”

Jerry says, “One?”

Marta responds, “Whatever you think,” with a look on her face that says this is going to be exciting. A student gasps, and Marta smiles.

Jerry repeats, “One?”

Another student gasps, “Ms. Marta!”

Marta tells the class that she sees different answers. “I want you to think about your answer. I want you to explain how you got that answer. Sidney tell me how you got your answer. Tell me how you got that number.”

Sidney says softly, “Just one.”

Marta asks, “Cooper how many did you circle?”

Cooper says excitedly, “Ten!”

Marta notes, “We have different answers. She circled one, he circled 10. But this says one? Cooper circled 10. Cooper, tell me why you circled 10.”
Cooper says, “I circled 10 because 10 plus four equals 14.”

Marta restates what Cooper said, as she writes on the board $10 + 4 = 14$. Then she asks Sidney why she circled one.

Sidney pauses, then says, “Because it’s there.”

“That would be perfectly okay except, I want you to notice something,” Marta says. “If I circle four and then I circle one, do I have a lot left over?” (As she says this she circles one circle and then one more.)

A student says. “No.”

Several other students object, “Yes.”

Marta says, “But how many did I write here?” (Circling the fourteen and tapping on the one in the tens place.) “What is the number?”

Several Students: “Fourteen.”

Marta: “Oh, oh, I see some looks. Remember these cards that we were looking at – (Taking out the 10 ten-frame cards used at the beginning of the lesson.) If I had 10 and then four more, I would have 14. It is the place that tells you the value. So, if I wrote… Now look at this, this is the reason I had you look at the number chart this morning. I was using the same number over and over again. One, two three, four, five, six, seven, eight, nine.” (She counts rapidly.) “Then we start again with another one…I have to figure out the place where it goes to show the value of it.”

She asks the students to clean their boards, so they can try it again. Marta has students write the number 11 in a box and draw 11 circles. Now she asks the students, “How many digits do I have?”

Ted: “One.”
Sam: “Two.”

Marta: “Two digits but they’re the same. But the place where they are will tell me the value. Let’s draw 11 circles. Now this time I would like you to circle this part of the number.” (Circling the one digit.) “Circle that amount.” (She surveys the class and circles one of the circles. Now she asks the students to circle the digit in the tens place.) “How many circles should I circle? Sidney?”

Sidney: “Ten.”

Marta: “We would circle 10 because of the spot where this one is. I’m going to teach you today about place value.”

The students erase their boards. Marta explains as she draws on her board, she draws 11 circles and circles one in it. Then she circles the other 10. Next, she draws what she calls a tower of 10. The tower of 10 is a tall narrow rectangle. She fills the tower with 10 circles and adds one circle outside the tower to represent 11.

Marta: “Now you see that 11 is one tower of 10 and one more.”

Cooper: “Oh, I get it!”

Marta: (Smiling) “Now you get it! Let me illustrate it a different way.”

Marta counts off 12 children and asks them to arrange themselves into ones and tens. “Arrange yourselves into a set of 10 and the left over,” she says. Then she sends the students back to their seats, and asks them to look in their material bags and sort the ones, the tens, and the big one, the hundreds. Using a document camera, she sorts counters into ones, tens, and hundred. She tells the students that they need to put their ones on the right side. On the left side, the students place the tens. Now she asks the students to make 32 using the base-10 counters.
She shows them two stripes of numbers zero through nine. She points out that although the numbers are the same numbers, and Marta points out that the value of the numbers changes depending on their placement. Using a tens and ones charts with slits, she slides number strips into the ones place and asks the students to show how much that is worth using base-10 blocks. After a short time, she asks the students to show her two cubes.

Next, she asks the students what would happen if she puts a one in the tens place, of the number chart, and asks the students to make that number. Marta walks around the class and sees what the students are doing. As she walks around she tells them to make a 12. When students have a problem, she tells them to make 12. When one student continues to have problems, Marta points to the chart telling him that he uses a tens block and two ones. To another child, she asks her how many tens she needs. Then Marta points to the one in the tens place of the chart. She points to the ones place of the chart and asks how many ones are needed.

Marta: “If you were writing the number 12 what number would you write first?”
Jared: “The one…”
Marta: “And then the?”
Jared: “The two.”
Marta: “Perfect. Why wouldn’t we write the two first?”
Arianna: “Because that would be twenty.”

Marta gives directions for students to complete a worksheet using what they have to help them. The students put away their manipulatives. Marta uses the document
camera to show the students what to do. The worksheet has pictures of base-10 blocks. The students are to choose the correct numbers to glue next to each number representation. She points to the tens block and asks how many tens. Then she points to the ones blocks and asks the students how many ones are there. After the students answer the question she asks the class how much is ten plus nine. The students can successfully answer her questions. Then she asks the students to count the blocks, while she points. They count 10, 11, 12, 13…19. The students proceed to write 19 in the box.

Marta: “Wilfred, what number do I write first?”

Wilfred: (Sounds unsure.) “One?”

Marta: “And how many ones are there?”

Wilfred: “Nine.”

Marta: “Nine ones. And that makes what, Parker?”

Wilfred: “Nineteen.”

Now she instructs the students to finish the worksheet. Marta goes to help a student. The student counts by one. Marta asks him how many tens and how many ones he has. She moves from table to table checking the student work. When student work is correct, she provides a compliment, “Perfect.”

She gathers the class’ attention telling them to clean up and return to the carpet before working on their journal. This process takes 11 minutes. Marta asks the students what they learned during today’s lesson. She calls on a hesitant student who answers, “Tens?”

Marta: “The power of tens. Did you notice that I had to have 10 of these (pointing to the separate blocks) to make a tower of ten?”
Christina: “Ones.”

Marta: “The difference between ones and tens.”

Alice: “Digits.”

Marta: “And digits; we discussed digits…The place where it stands is the value of the number. What does this thee mean?”

Marta continues a review and then helps the students read the journal assignment. The students go to their seats and Marta works with three students at her kidney shaped table. She calls on Parker, Sam, and Andy to review today’s lesson. Using a hundreds chart she asks the boys to find 12 on the chart. Now she asks them to count out 12. Now she has them count ten from the 12 and separate them. Then they trade the 10 ones for a 10 block. She shows them on the number chart that the one means the tens block and the two single blocks are the ones. She repeats the procedure individually with the students at her desk. The class ends with Marta asking the students to put their journal away.

Table 9 describes the MQI strategies used in this lesson. Marta uses several strategies found within the Richness of the Mathematics dimension of the MQI. Linking Between Representations is a strategy used throughout the lesson. Marta used pictorial ten-frames and a place-value chart in this lesson. She continually drew students’ attention explicitly comparing how each representation showed the same information. In most cases during the lesson her explanations were clear. The students make 14 circles and separate the ones and the tens by circling the four ones. In most lesson segments Marta uses correct mathematical vocabulary, ones and tens. There is an instance where she uses the word “number” when “digit” would have been appropriate. The dimensions of Working with Students and Mathematics and Common Core-Aligned Student Practices
Table 9

MQI Analysis for Observation Four - Grade One

<table>
<thead>
<tr>
<th>MQI Dimensions</th>
<th>Segments</th>
<th>Scores</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>RICHNESS OF THE MATHEMATICS</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Linking Between Representations</td>
<td>S1</td>
<td>N</td>
<td>Marta links the meaning of a quantity of ten to representations of ten using base-10 blocks and a base ten numeration system where the value of a digit changes depending on its place.</td>
</tr>
<tr>
<td></td>
<td>S2 – S6</td>
<td>H</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S7</td>
<td></td>
<td></td>
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<tr>
<td></td>
<td>S8</td>
<td>H</td>
<td></td>
</tr>
<tr>
<td>Explanations</td>
<td>S1</td>
<td>N</td>
<td>The teacher clearly explains place value to students.</td>
</tr>
<tr>
<td></td>
<td>S2 – S6</td>
<td>H</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S7</td>
<td>na</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S8</td>
<td>N</td>
<td></td>
</tr>
<tr>
<td>Mathematical Sense-Making</td>
<td>S1</td>
<td>N</td>
<td>Students and teacher develop the concept that digits mean different things depending on their placement in our written numeration system.</td>
</tr>
<tr>
<td></td>
<td>S2 – S6</td>
<td>H</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S7</td>
<td>na</td>
<td></td>
</tr>
<tr>
<td>Multiple Procedures or Solutions Methods</td>
<td>S1 – S6</td>
<td>N</td>
<td>Students only use one solution method.</td>
</tr>
<tr>
<td></td>
<td>S7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Patterns and Generalizations</td>
<td>S1 – S3</td>
<td>H</td>
<td>Students provide generalizations to describe part-part-whole when finding the missing part using a ten frame.</td>
</tr>
<tr>
<td></td>
<td>S4</td>
<td>N</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S5</td>
<td>N</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S6</td>
<td>H</td>
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<tr>
<td></td>
<td>S7</td>
<td>na</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S8</td>
<td>H</td>
<td></td>
</tr>
<tr>
<td>Mathematical Language</td>
<td>S1 - S6</td>
<td>H</td>
<td>Teacher provides and uses mathematical vocabulary.</td>
</tr>
<tr>
<td></td>
<td>S7</td>
<td>na</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S8</td>
<td>H</td>
<td></td>
</tr>
<tr>
<td><strong>WORKING WITH STUDENTS AND MATHEMATICS</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Remediation of Student Errors and Difficulties</td>
<td>S1 – S5</td>
<td>N</td>
<td>Teacher remediation is not necessary because students did make any mistakes.</td>
</tr>
<tr>
<td></td>
<td>S6</td>
<td>L</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S7</td>
<td>na</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S8</td>
<td>H</td>
<td></td>
</tr>
<tr>
<td>Teacher Uses Student Mathematical Contributions</td>
<td>S1</td>
<td>H</td>
<td>Teacher uses student contributions, but student contribution is answering close-end questions.</td>
</tr>
<tr>
<td></td>
<td>S2 – S5</td>
<td>L</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S6</td>
<td>H</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S7</td>
<td>na</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S8</td>
<td>H</td>
<td></td>
</tr>
<tr>
<td>MQI Dimensions</td>
<td>Segments</td>
<td>Scores</td>
<td>Notes</td>
</tr>
<tr>
<td>----------------</td>
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<td>--------</td>
<td>-------</td>
</tr>
<tr>
<td><strong>COMMON CORE-ALIGNED STUDENT PRACTICES</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Student Provide Explanations</td>
<td>S1</td>
<td>N</td>
<td>Students explain the difference between the value of a digit in the tens place and a digit in the ones place.</td>
</tr>
<tr>
<td></td>
<td>S2</td>
<td>M</td>
<td></td>
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<tr>
<td></td>
<td>S3</td>
<td>N</td>
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<tr>
<td></td>
<td>S4</td>
<td>N</td>
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<tr>
<td></td>
<td>S5</td>
<td>N</td>
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<tr>
<td></td>
<td>S6</td>
<td>H</td>
<td></td>
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<tr>
<td></td>
<td>S7</td>
<td>na</td>
<td></td>
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<tr>
<td></td>
<td>S8</td>
<td>N</td>
<td></td>
</tr>
<tr>
<td>Student Mathematical Questioning and Reasoning</td>
<td>S1 – S6</td>
<td>N</td>
<td>Marta scaffolds student thinking. There is no evidence of student reasoning or questioning.</td>
</tr>
<tr>
<td></td>
<td>S7</td>
<td>na</td>
<td></td>
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<tr>
<td></td>
<td>S8</td>
<td>N</td>
<td></td>
</tr>
<tr>
<td>Student Communicate about the Mathematics of the Segment</td>
<td>S1</td>
<td>H</td>
<td>Students communicate concepts with single word answers.</td>
</tr>
<tr>
<td></td>
<td>S2</td>
<td>H</td>
<td></td>
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<tr>
<td></td>
<td>S3</td>
<td>L</td>
<td></td>
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<tr>
<td></td>
<td>S4</td>
<td>H</td>
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<td>S5</td>
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<td>S6</td>
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<td></td>
<td>S7</td>
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<td>S8</td>
<td>L</td>
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<tr>
<td>Task Cognitive Demand</td>
<td>S1</td>
<td>L</td>
<td>Students discuss the difference between a 1 in the tens place and a 1 in the ones place.</td>
</tr>
<tr>
<td></td>
<td>S2</td>
<td>H</td>
<td>Other times Marta provides students with procedures to solve the problems.</td>
</tr>
<tr>
<td></td>
<td>S3</td>
<td>H</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S4</td>
<td>L</td>
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<tr>
<td></td>
<td>S5</td>
<td>L</td>
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<td></td>
<td>S6</td>
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<td></td>
<td>S7</td>
<td>na</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S8</td>
<td>L</td>
<td></td>
</tr>
<tr>
<td>Student Work with Contextualized Problems</td>
<td>S1</td>
<td>N</td>
<td>In most segments, the students use their knowledge of counting to understand the base ten system of writing numbers.</td>
</tr>
<tr>
<td></td>
<td>S2 – S6</td>
<td>M</td>
<td></td>
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<tr>
<td></td>
<td>S7</td>
<td>na</td>
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<tr>
<td></td>
<td>S8</td>
<td>M</td>
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</table>
are not used consistently at a high level of quality. A detailed summary analysis for Observation Four can be seen in Table 9.

**Observation Five - Grade Two**

My next observation of Iris occurred the following year. She had a student teacher; who was providing instruction for the first-grade class, while she instructed the second-grade class for math. Iris is conducting her math class in the aftercare room. This group is composed of the entire second-grade class. The students seem very happy to engage in the math activity.

The class starts with a review. She tells the students that they will see a subtraction problem and they must solve it using mental math. She writes \(17 + 30 = \), and has the students chorally read the equation. She asks them what is the term used to describe the number they are looking for; after the answer “the sum”, she asks them to think about what they must do to solve the problem using mental math.

She asks a student to describe his process for solving the problem. He says that you must add your ones and then your tens to solve the problem. Iris then asks him what he got when he used that process. He answers “47.” She than asks students if anyone solved the problem a different way.

Kelly: “You look at the second two numbers and you add them somewhere in your head. Then you go to the first two numbers. And then you have the answer.” Iris notices that Kelly has the kernel of the idea but she doesn’t quite know how to explain it. So, she pushes her to be more precise with her language. “When you say first two numbers, (pointing the equation on the board,) which of these numbers are you talking about? Do you mean you add the one and three first?” Kelly sounds appalled, “No, you
add the ones first.” Iris continues to question her, “So you add the zero and the seven, then you add the one and three? What do you call the one and three?”

Kelly seems to know lots of mathematical vocabulary and in this situation, she is not sure what word to use. First, she says they are addends. When Iris asks Kelly what other word could be more specific about what you call the one and three, a chorus of children answer “tens.”

When Iris asks the students what other way can you do this; another child explains that lining the numbers up vertically could solve the problem. Iris rewrites the problem and the students solve it. Iris asks them to confirm if they got the same answer both ways.

Next, they do a subtraction problem, $81 - 20 = $. Iris asks the students to read the problem aloud. Some of the students say, “take away”, while others say “minus.” Again, this is an instructional moment. She asks them about the minus sign, what is another way to say this? Some student says subtract, Iris pushes their answers so they use the words more exactly. With subtraction, the students seem unsure of themselves. They are quiet. Iris waits as they process the problem. When she asks for solutions again she focused on ‘how did you do it’, not ‘what is the correct answer.’ The students explain subtracting the ones place, then the tens place. They all agree that they could have thought about it vertically to solve the problem.

Iris stretches their thinking by asking them for a way to use pictures. She gives them a new problem, then asks the students how they could use quick pictures to solve the problem. Iris writes, “$54 - 20 = ?$” on the white board. Kelly draws on her board, five lines corresponding to tens and four boxes corresponding to ones. Then for a
moment she seems stuck on what to do next. Her classmates want to help her, but Kelly insists that she can do it herself. After a moment of thought, she writes 4 below the four squares, circles two sticks and crosses them out and writes 3. The other students are asked to confirm if Kelly has done the problem correctly. Once they have checked her work the class moves on.

Iris tells the students that they will continue to work on subtraction today, but they will, “kick it up a notch.” She tells them they will be subtracting numbers with a zero in the middle. A trickle of nervous laughter runs through the students.

She gives them a word problem. “I have 504 cars in the parking lot. During the day 173 cars leave. So how many cars do I have left in the parking lot?” First, she gets them to figure out what operation they need to perform, “What are we trying to figure out in this story problem? What’s the question, not the answer? What’s the question?”

Several students are eager to answer the question. Sofia says that the problem says take away. When prompted, “Does it say take away?” She explains further, “It says 173 leave,” With an emphasis on the leave to show that leave is the same as take away.

Another student says that you know it’s subtraction because the problem says the bigger number first. Iris counters, “Does that rule always work?” Another student points out that the problem could have had some cars entering the garage.

She repeats the word-problem question then asks, “What words in the problem make this a subtraction problem?” The same student says, “How many cars did not leave.” At this point, Iris decides to move on.
She asks the student to identify what’s in the problem that they haven’t seen before. As a group they answer, “The zero in the tens.” She tells them that they need to figure out how to do that type of problem.

Using base-ten blocks they set up the problem written on the board, 504 – 173 =. Colin places 5-hundreds flats, and four cubes representing the ones place. Iris asks why there are not tens rods in the number. They all answer, “Because there are no tens in the number.” Now they proceed to solve the problem with the blocks.

Teacher: “Can we take three away from the four?”
Students: “Yes.”
Teacher: “What are left with here?”
Students: “One.”
Kelly: “One hundred.”
Teacher: “Is it a hundred?”
Kelly: “No it’s a one.”
Teacher: “Do we have any tens here? (Pointing to the board.) How are we going to take 7 tens away?”
Cynthia: “Go to the hundreds”
Teacher: “Okay, Sofia said go to the hundreds. So, we are going to go the hundreds and what are we going to do?”
Cynthia: “Take it away.”

Iris asks another student to show her what must be done. Then she points out, “Do we take that hundred away…and put it away [meaning back in the box]? What do we do with this hundred?”
Student: “Trade it for tens.”

Teacher: “How many do we need to trade it for?”

Teacher: (After Brandon counts-out 10 tens.) “Do you understand what Brandon did?”

Students “He traded.”

Teacher: “What is another word we use to describe what Brandon did? (Students seem stumped.) Re…?”

Students “Regroup!”

Teacher: “Okay, let’s make sure. How many did we break this into?” (Holding the hundreds flat.) “Let’s count by tens.”

Everyone: “10, 20, 30, 40, 50, 60, 70, 80, 90, 100.”

Teacher: “So this one (Holding the hundreds base-10 flat.) “we don’t put back down there because we have the same amount. We just traded it, regrouped it.”

Teacher: “Now can we take 70 away? Morgan, will you take them away?” (After Noelle hands her the 7 tens. Iris puts them away.) “These are going away because we took them away.”

Teacher: “How many tens do we have now?”

Students: “Three tens.”

Teacher: “How many hundreds do we have now?”

Iris writes four and the students quickly correct her that they need to take one away.

Teacher: “We started with five hundreds base-10 flats then we traded one hundred. Now we need to take a hundred away.”
Kelly: (In a strong voice.) “We already took a hundred away.”

Kevin: “We need to take another one, because we regrouped the other one. We didn’t subtract it.”

Kelly is satisfied with Kevin’s explanation and she places her hand over Iris as she writes the number three, as if to say I agree with you now. But another voice speaks up, “How come we had to take another one away?” This time Iris tries to explain it using a different method.

Teacher: “Let’s write the problem the vertical way and write everything we do with the blocks as we do it.” Iris writes 504 – 173 = as a vertical algorithm and places base-10 hundreds flats and four base-10 ones’ cubes. Then she regroups with a hundreds flats and writes it on the vertical algorithm. Next, she trades the one of the hundreds base-10 flats for 10 tens-base 10 rods, and writes the regrouping on the algorithm. Now she begins to subtract. She takes three from the four blocks and writes one in the algorithm. She takes seven base-10 rods away, and writes three in the algorithm. Finally, she takes one hundreds base-10 flat, and writes three in the algorithm.

By solving the problem again and looking at the algorithm, all the students were satisfied that the answer was correct. Iris repeated the process with another similar problem this time grouping across zero to solve 605 – 267 =. Using the algorithm and the base-10 blocks the students solved the problem just like they did the previous one. As they solve the problem with the base-10 blocks they write the numbers on the board. After the problem is solved, Iris tells the students that they will do another problem using the blocks. There is a united groan by the students. Iris reminds them that attitude is important. The students enthusiastically continue to solve the next problem. Iris took time
to remind the students that they need to think each time they subtract, “Do I have enough to subtract from?” With that the lesson is over. Because I’m there filming, the students say, “Cut!”

This lesson shows strong use of the Richness of the Mathematics strategies. The teacher and students spent considerable time comparing how subtracting 504 – 173 is the same when using base-ten blocks or the subtraction algorithm with regrouping. Teacher questioning guided students in sense making. Using base-10 blocks, Iris had students demonstrate how to subtract with regrouping. The students explained their thinking as they worked through the problem. The students subtracted using regrouping. In the Working with Student and Mathematics dimension, the teacher uses student contribution effectively. Students give detailed explanations of their mathematical rationale. Using the Common Core-Aligned Student Practices, Iris provides the students with ample opportunity to Communicate about the Mathematics of the segments, provide explanations, and show mathematical reasoning. A detailed summary analysis for Observation Five can be seen in Table 10.

Observation Six - Grade Three

Maria teaches a self-contained third-grade class with 20 students. The students seem to have internalized classroom procedures. She starts the math class by reminding the students that they know the procedure for getting out their math materials, then asks them to get their math boards and math book “quietly and quickly”. She plays a mathematics song as the students get their supplies and she passes out bags with counters. She carefully tells the students where to put their counters, math boards, math books, and markers. Once everything is organized she starts the lesson.
### MQI Analysis for Observation Five - Grade Two

<table>
<thead>
<tr>
<th>MQI Dimensions</th>
<th>Segments</th>
<th>Scores</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>RICHNESS OF THE MATHEMATICS</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Linking Between Representations</td>
<td>S1 – S5</td>
<td>H</td>
<td>Iris used base-ten blocks, quick pictures and digits to represent the base ten system.</td>
</tr>
<tr>
<td>Explanations</td>
<td>S1 – S5</td>
<td>H</td>
<td>Students provided complete explanations of mathematical ideas.</td>
</tr>
<tr>
<td>Mathematical Sense-Making</td>
<td>S1 – S5</td>
<td>H</td>
<td>Students explored relationships between hundreds, tens and ones.</td>
</tr>
<tr>
<td>Multiple Procedures or Solutions</td>
<td>S1 – S5</td>
<td>H</td>
<td>Students solved subtraction problems with regrouping across a zero using concrete objects, pictures and subtraction algorithm.</td>
</tr>
<tr>
<td>Methods</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>Patterns and Generalizations</td>
<td>S1 – S5</td>
<td>H</td>
<td>Students developed generalized rules for subtraction with regrouping across zeroes.</td>
</tr>
<tr>
<td>Mathematical Language</td>
<td>S1 – S5</td>
<td>H</td>
<td>Teacher and students used mathematical vocabulary to explain their thoughts.</td>
</tr>
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<td></td>
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<tr>
<td><strong>WORKING WITH STUDENTS AND MATHEMATICS</strong></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>Remediation of Student Errors and</td>
<td>S1</td>
<td>N</td>
<td>Teacher addressed student difficulties when present.</td>
</tr>
<tr>
<td>Difficulties</td>
<td>S2</td>
<td>M</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S3</td>
<td>H</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S4</td>
<td>N</td>
<td></td>
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<tr>
<td></td>
<td>S5</td>
<td>N</td>
<td></td>
</tr>
<tr>
<td>Teacher Uses Student Mathematical</td>
<td>S1 – S5</td>
<td>H</td>
<td>Teacher used student ideas to move mathematical understanding forward.</td>
</tr>
<tr>
<td>Contributions</td>
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<tr>
<td><strong>COMMON CORE-ALIGNED STUDENT PRACTICES</strong></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Students Provide Explanations</td>
<td>S1 – S5</td>
<td>H</td>
<td>Students formed conclusions based on patterns discussed.</td>
</tr>
<tr>
<td>Student Mathematical Questioning and</td>
<td>S1 – S5</td>
<td>H</td>
<td>Students offered explanations and discussed solution methods. S1</td>
</tr>
<tr>
<td>Reasoning</td>
<td></td>
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</tr>
</tbody>
</table>
Table 10—Continued

<table>
<thead>
<tr>
<th>MQI Dimensions</th>
<th>Segments</th>
<th>Scores</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Students Communicate about the Mathematics of the Segment</td>
<td>S1 – S5</td>
<td>H</td>
<td>Students formed conclusions based on patterns discussed.</td>
</tr>
<tr>
<td>Task Cognitive Demand</td>
<td>S1 – S5</td>
<td>H</td>
<td>Students were engaged in the mathematical task in these segments.</td>
</tr>
<tr>
<td>Student Work with Contextualized Problems</td>
<td>S1</td>
<td>N</td>
<td>There were instances where students did not work on problems that required them to apply mathematical skills to situations.</td>
</tr>
<tr>
<td></td>
<td>S2</td>
<td>N</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S3 – S5</td>
<td>H</td>
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</tbody>
</table>

Maria reminds the students what was covered yesterday, then introduces today’s topic using a fraction to tell how many parts are in a group. Using counters, Maria instructs the students to find one-third of 15 in their desk groups. She repeats her instructions asking them to work together and figure out what you would do to find one-third of 15.

She walks around the room helping students get started. She asks, “So what would you do to start.” I can’t hear what the student says, but Maria restates, “So you count the counters. How many are you going to count?” Again, the child’s voice is muffled. Maria restates, “15. Where did you get that number?” The student’s answer is not clear. As she walks away she tells them, “You got 15. Now work together to find the answer,” as she walks away toward another group.

When she reaches the group, she asks, “What are you guys doing?” The children explain what they are doing. And she asks them how many groups they have; then, how many are in each group. She nods at their answer and walks away to another group.
“What did you guys do?” she asks the next group. She listens carefully and restates what they said, “Five in each circle.” She asks if the students know what 1/3 of 15 is. Then she nods her head and moves on. She asks the next group, “What did you guys do?” She nods as they answer. “You put five in each group and how many groups do you have? Okay, so how would you show one-third?” She asks the next group why they put five in each group. They explain that three times five is 15.

Maria calls the class’s attention, “Class, Class.” They respond, “Yes, yes.” She asks, “Shelby what was the first thing you had to do?” She rephrases Shelby’s answer, “You laid out your groups. How many groups do you have?” She continues this procedure, “Three groups. Where did you get the idea to use three groups?” Emma answers, “Because I think we were talking about that yesterday.” Maria accepts Shelby’s answer and asks Sally to add to what Shelby has said. When Sally is unable to answer, she repeats the question to Wilber. Wilber does not sound confident, “I think five divided by three. No, I mean 15 divided by three is five so we put five in each of the three groups.”

Maria smiles as Wilber answers, and further asks, “Yes I see that other groups have put five counters into three circles. But where did we get the idea that we needed three groups? Sophie.” Sophie answers, “Because five plus five plus five equals 15 groups.” I can’t hear the first student’s answer.

Maria tries to refocus the students to the facts found in the word problem. She rereads the word problem, and then asks, “What in our instruction made us think we needed three groups? What is our denominator in our fraction?” Ivan answers, “Three?”
Maria replies, “Do you think that that has something to do with why we chose three groups? Emma. Well so three goes into 15.”

“Can anybody answer the question in the problem? What is one-third of 15?” Emma continues, “Five counters.” Maria probes further, “How do you know that?” “Because there are five counters in each group,” Emma explains.

Maria asks the students to get their markers and math boards and reads the next problem from the board, “Audrey buys a bouquet of 12 flowers. One third of them are red. How many of the flowers are red?” After she read the problem, she helps the students identify the important facts found in the problem, “What would your first step be in our fantastic five?” “Find the facts,” replies Emma. Maria probes further, “What are the facts, what do we know?”

One student says there are 12 flowers, and another asks what is a bouquet? “What’s another fact that we know?” A student answers but it’s hard to hear. Maria’s responds, “Yes, one-third of the flowers are red. Are there any more facts? Do we have a question? What is the question?” Maria circles the question as she reads it, “How many of the flowers are red?”

“Now we need to think about how we’re going to use our supplies to solve this problem. So, the supplies that you have on your desk; what can you do to use your supplies to solve your problem?” Kelly answers her teacher, “Make three groups.” Maria follows up, “How would you make three groups, Kelly?” Kelly continues, “Draw three big circles. Maria counters, “Kelly, why are we making three groups?” “Because the problem says one-third,” Kelly answers.
“What would your next step be?” asks Maria. Addie asks a question. She doesn’t understand what do. Maria helps her out by guiding her to understand that the chips are used instead of flowers to solve the problem. Addie says she needs 12 chips to represent the 12 flowers. Once Addie understands Maria moves on with the lesson by reminding the students that Kelly said they need three circles because they are looking for one-third and Audrey said we need 12 chips because we have 12 flowers. “Now that we have our supplies ready we can solve the problem,” says Maria.

Maria: “Now what is our next step? Benjamin?”

Juan: “You put four in each group.”

Maria: “Why would you put four in each group?”

Juan: “It’s because there are 12 flowers and four plus four plus four equals 12.”

Maria: “So you’re using repeated addition to make them into equal groups?”

Maria has all the students make three circles on the personal white boards and place four chips in each circle, as she places chips on the board for students to see. Then she asks the students, “How are we going to find out what one third is? Shelby?”

“What I did is put two groups of yellow counters and one group of red counters. Then I knew that one group was red was the answer,” replies Shelby. Maria restates Shelby’s answer, and then she asks the students to circle one of the groups to show one-third.

She walks around checking how students solved the problem. Bonnie has four groups. Maria asks, “How many groups do we need?” This question helps the students regroup their chips into three groups. She asks the class how many one-third of 12 is.
After Thomas answers four, Maria asks, “And why do you know it’s four, Thomas?” Thomas’ responds with four times three is 12.

Maria turns to the class and asks, “How is finding one-third of 12 like division?” Emma answers, “We know the whole but we need to find the parts.” Maria asks for another student to add to Emma’s comment. Audrey answers, “If you divide the 12 among three groups it’s going to have four in each group.” As each student answers her question, Maria’s body language is encouraging and positive.

Maria tells the class that they will continue learning math in their groups. The students seem to be divided into three ability groups: one group works on fluency, one group works on an assignment in their math textbook, and one group works with Maria. She is careful to give very precise directions on how each group should proceed to their area.

Maria starts her group instruction. She uses the problems in the grade three Go Math textbook. The students have been taught a procedure for sitting on the carpet and having their textbooks and math boards available and organized. She comments on the students who are ready to begin instruction.

Maria asks one student to read the problem. After the problem is read, she asks the students to use their problem-solving strategy “Fantastic Five” to dissect the problem. First, she asks the students to identify the facts. Isabelle identifies that there are 12 flowers, and Sophie identifies that one-fourth of them are purple. Next, she asks the students to identify the next step in “Fantastic Five”. Sophie reads the question. Maria asks the students what they should do next. Sophie answers, “We can use our circles and
counters.” Maria asks the student to use their boards, markers, and chips to model the problem.

Toby asks, “How many circles do we draw?” Maria praises Toby for the question and asks the students to answer his question. When no student answers the question, Maria coaches them to reread the problem and identify the facts. Diana answers, “One-fourth.” Maria looks quite quizzically at Ariel, “Do we draw one-fourth of a circle?”

Ariel: “No. four circles.”

Maria: “Why do we draw four circles?”

Ariel: “Because it’s one of four.”

Maria: “Siena has moved on to the next part, solving the problem. What did you do next?”

Ariel: “Put the chips in the circle.”

After watching the students work, Maria asks them to explain how they used the model to find the answer. A student states that you must count one group of the counters. Maria counters by asking why only one group. If the students give partial answers Maria probes their ideas to help them clarify their thoughts. Several students answer the question that one group is the answer. Maria keeps pushing them to articulate why that is the correct answer.

“What facts were in our problem that helps know that the answer is one group?” asks Maria. Finally, one student says, “Because it’s one-fourth and that’s one group it’s not two-fourths.” Maria probes further, “How many groups would be counted if it was two-fourths? Rebecca answers, “Two groups.” Maria commends her for the correct answer.
Moving on to the next problem, the students are asked to find one-half of eight using the given picture model. Maria asks the students to describe how they can use the model to solve the problem. She asks, “How many counters are you given?”

Nancy: “Make them into two equal groups.”

Maria: “Oh! Nancy said make them into two equal groups. What part of that fraction told you that you needed two equal groups?”

Walter: “The denominator.”

Maria: “Good. How are we going to find out what one-half of eight is?”

Walter: “One circle.”

Maria: “Great! Now do two through four on your own.” As the children work she helps students that need help by asking them questions about the decisions they made. When she sees that a student group has solved the three problems, she asks them to go on to the next page.

The next page has word problems. One student reads the problem aloud. Maria asks about the “Fantastic Five.” She repeats the procedure asking the students to identify facts, identify the question, pick a modeling strategy to solve their problem, and then she asks the students to work on their own to solve the problem. She sits beside some students quietly asking them questions, which seems to help them refine their problem-solving skills. She keeps emphasizing “why did you do that?” After they finish, she gives the students an assignment, and then they return to their seats.

Maria calls the next group to the table and they open their books to the same pages as the other group. As with the other group, Maria asks someone to read the word problem aloud. After the problem is read, she asks the student to use the “Fantastic Five”
problem-solving heuristics to solve the problem. After the students have a chance to work on the problem, she asks the students to explain their process and rationale for their mathematical choices.

Maria continually asks why, what, and how questions. Maria keeps focusing the students using why questions, “Why did you make that choice? Why did you do that? Can you explain why you did (blank)?” Paired with her why questions, Maria asks students what questions such as “What did you do?” She always restates the answers students give. Maria’s use of how questions connects what students did with why they did it, “How did you use your model to help you solve the problem?” When she restates their answers she also polishes their ideas while asking, “Is this what you meant to say?”

This group is almost independent. Maria confers with several students. She reminds the students to use their problem-solving heuristic. She reminds students to use a model for each problem while she is supervising the rest of the class. After ten minutes, this group returns to the rotation and she calls the next group to her table. Students in this group are asked to give a rationale for their answers. Maria notes that some of the students have answered the problems but have not used the “Fantastic Five” heuristic. When they complete the problems, she moves them to the floor using counters, the students work on the Higher-Order-Thinking-Skills problems.

This last group seems to be the least mathematically skilled of all the groups. This group is sitting on the floor and solving problems using manipulatives, while the other groups solved the problems using only pictorial representations. Maria continues to ask the same types of questions, probing students to understand each other’s thinking processes.
For the last part of the math lesson, Maria brings all the students together. Maria sets up a problem, “Tomorrow at our pool party, if all of us are going, how many of us are going?” Using the counter, students were asked to divide the class into groups, who will ride in two buses. The students are instructed to work together at their tables. Once the groups are finished, she asks the class to identify what is the fraction of students on each bus. This takes the same problem but reverses the question. She walks around the room asking each group what fraction they have. The first group she asks says the one is the fraction. Maria reminds the group that one is a whole number. The one groups answers, “one/eleven”. Another two groups say, “eleven/twenty-two”. Maria does not tell the groups whether they are right or wrong. She returns to the front of the room, and after a thoughtful moment she asks one group to identify the denominator. After she gets the answer of 22; she asks for the numerator, and gets the answer, “eleven/twenty-ones”. She continues to walk to each table asking students, “What fraction did you get?” Her answers are noncommittal. Finally, she calls the groups attention back to the front of the class. She asks Adam, “What did your group get for the denominator?” He answers, “Twenty-two.” She follows up with a second question, “Where did you get the number?” He responds that there are 22 students in the class. She asks another student the same two questions about the numerator. Without further discussion, she asks the class to put away their math material and books.

Maria uses questioning to guide students’ ability to make sense of the mathematical concepts they are learning. The questions usually begin with “why”. “Why would you put four in each group? Why did you make that choice?” Other questions begin with how and where. “How did you get that number? How did you get that answer?
Where did you get the idea to use three groups?” She uses “what” questions to make students explain their thinking. “What in our instruction made us think we needed three groups? What are the facts we know?” The questions she uses draw the students’ attention to links between representation, explanations of concepts, and making sense of numbers they are using to solve the problem. This questioning pattern makes the lesson strong with the strategies found in Richness of Mathematics and Common Core-Aligned Student Practices. Her carefully structured questions don’t provide many opportunities for students to be confused. There is very little student remediation needed. A summary analysis for Observation Six can be seen in Table 11.

**Observation Seven - Grade Four**

Deborah has taught fourth-grade for over 10 years. Her previous students describe her as a teacher who loves them and demands that each student do their best. This group of fourth-grade students is busy getting ready to begin math rotations. Students spread throughout the room working on math facts with a partner, writing in their math journal or completing a previous assignment. One group of students is meeting with the teacher at a kidney shaped table. Overall, there is a rushed feeling about these math lessons.

**Small Group Instruction – Group One**

As the students settle in to their places, Deborah asks if anyone has a question about the assignment of the previous day. One student didn’t get problems seven and eight in yesterday’s practice. Deborah reads the problem aloud, “Jason’s teacher asks him to color four eighths of his grid. He must use three colors: red, blue, and green. There
Table 11

MQI Analysis for Observation Six - Grade Three

<table>
<thead>
<tr>
<th>MQI Dimensions</th>
<th>Segments</th>
<th>Scores</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>RICHNESS OF THE MATHEMATICS</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Linking Between Representations</td>
<td>S1</td>
<td>N</td>
<td>Throughout most of the segments Maria went back and forth between symbolic and notational representations.</td>
</tr>
<tr>
<td></td>
<td>S2 – S8</td>
<td>H</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S9</td>
<td>N</td>
<td></td>
</tr>
<tr>
<td>Explanations</td>
<td>S1</td>
<td>N</td>
<td>Maria used questions to guide students to their own explanations.</td>
</tr>
<tr>
<td></td>
<td>S2 – S9</td>
<td>H</td>
<td></td>
</tr>
<tr>
<td>Mathematical Sense-Making</td>
<td>S1 – S9</td>
<td>H</td>
<td>Instruction leads students to explain the meaning of fractional numbers.</td>
</tr>
<tr>
<td>Multiple Procedures or Solutions</td>
<td>S1</td>
<td>N</td>
<td>Teacher provided opportunities for students to explain multiple ways they had come up with the answers.</td>
</tr>
<tr>
<td>Methods</td>
<td>S2</td>
<td>L</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S3</td>
<td>H</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S4</td>
<td>N</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S5</td>
<td>N</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S6 – S8</td>
<td>M</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S9</td>
<td>L</td>
<td></td>
</tr>
<tr>
<td>Patterns and Generalizations</td>
<td>S1</td>
<td>N</td>
<td>There were some instances where Maria emphasized the commonalities between problem solutions. This helped students articulate generalizations.</td>
</tr>
<tr>
<td></td>
<td>S2</td>
<td>L</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S3</td>
<td>H</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S4</td>
<td>N</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S5</td>
<td>N</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S6 – S8</td>
<td>M</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S9</td>
<td>L</td>
<td></td>
</tr>
<tr>
<td>Mathematical Language</td>
<td>S1</td>
<td>N</td>
<td>Although students seemed to know the words denominator and numerator, specific mathematical language was not used consistently.</td>
</tr>
<tr>
<td></td>
<td>S2</td>
<td>H</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S3</td>
<td>H</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S4</td>
<td>M</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S5</td>
<td>H</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S6 – S9</td>
<td>M</td>
<td></td>
</tr>
<tr>
<td>WORKING WITH STUDENTS AND MATHEMATICS</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Remediation of Student Errors and</td>
<td>S1 – S3</td>
<td>M</td>
<td>Teacher addressed student confusion and lack of understanding with questions that asked to combine what they already understood about fractions with the current concepts they were working with.</td>
</tr>
<tr>
<td>Difficulties</td>
<td>S4</td>
<td>L</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S5</td>
<td>L</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S6</td>
<td>N</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S7</td>
<td>N</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S8</td>
<td>H</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S9</td>
<td>N</td>
<td></td>
</tr>
</tbody>
</table>
Table 11—Continued

<table>
<thead>
<tr>
<th>MQI Dimensions</th>
<th>Segments</th>
<th>Scores</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Teacher Uses Student Mathematical Contributions</td>
<td>S1 – S9</td>
<td>H</td>
<td>Maria consistently used questions to elicit student understanding.</td>
</tr>
</tbody>
</table>

COMMON CORE-ALIGNED STUDENT PRACTICES

<table>
<thead>
<tr>
<th>Student Provide Explanations</th>
<th>S1 – S8</th>
<th>H</th>
<th>Maria’s questions framed student explanations of mathematical concepts.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Student Mathematical Questioning and Reasoning</td>
<td>S1 – S8</td>
<td>H</td>
<td>Students reasoned comparing representations and problem-solving methods.</td>
</tr>
<tr>
<td></td>
<td>S9</td>
<td>M</td>
<td></td>
</tr>
<tr>
<td>Student Communicate about the Mathematics of the Segment</td>
<td>S1 – S8</td>
<td>H</td>
<td>There was a lot of student communication during most segments.</td>
</tr>
<tr>
<td></td>
<td>S9</td>
<td>M</td>
<td></td>
</tr>
<tr>
<td>Task Cognitive Demand</td>
<td>S1</td>
<td>M</td>
<td>Tasks demanded that students think and use multiple representations and explain their problem-solving choices.</td>
</tr>
<tr>
<td></td>
<td>S2 – S8</td>
<td>H</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S9</td>
<td>M</td>
<td></td>
</tr>
<tr>
<td>Student Work with Contextualized Problems</td>
<td>S1</td>
<td>N</td>
<td>Students used their knowledge of numbers to develop solutions to find fractions of a given whole number.</td>
</tr>
<tr>
<td></td>
<td>S2 – S9</td>
<td>H</td>
<td></td>
</tr>
</tbody>
</table>

must be more green sections than red sections. How can Jason color the sections of his grid to follow all the rules?”

Deborah asks the student what she would do. The student replies, “four eighths equals two eighths plus two eighths.” Deborah asks the student how many colors they must use. The student answers with a glint of understanding, “Oh! It has to be three of them. He could do four eighths equals one eighth plus one eighth plus two eighths.” Then Deborah pushes the student to now give color to the fractions and reminds the student that there must be more green sections than red sections. After a slight pause, she asks,
“Where would you place the color green?” The student says she would make two eighths green, and two eighths each red and blue.

Another student asks, “On number four it says, ‘Which of these is a prime number?’” All the students answer that the number is a prime number. Another student adds, “One and 11 are prime numbers.” Deborah reminds the students that prime numbers are greater than one. The student’s confusion continues, “I have 21 as a prime number.” Deborah asks the group if 21 is a prime number. Several students respond “No” and one student says “three times seven equals 21.” Deborah next reviews each number in the list asking the students if it is prime or not.

During the next few minutes, Deborah reads the correct answers to yesterday’s set of practice problems’ then asks students for their correct answer. One student asks about a problem where his answer was different from the answer Deborah gave. She determines that the answer he had is also correct and moves on. Other students have the same question. They give their answers, she determines if they are correct, and moves on to the next student. Once the grading is complete, each student says how many problems he or she got correct out of 13.

Moving on, Deborah tells the students to open their books to the next lesson, 7.3, Adding Fractions Using Models. The students prepare for the lesson by opening their books and taking out fraction strips.

Teacher: “We’ve been working with fractions with like denominators. Can someone tell me what is 4/8? Arnold, can you tell me what the four is?”

Arnold: “The numerator.”

Teacher: “Can you tell me what does the numerator name?”
Arnold: “Parts of the whole.”

Teacher: “It tells you how many parts of the whole. Okay. What about the eight? What does the eight represent?”

Several Students: “The denominator.”

Teacher: “And what does the denominator identify in the fraction?”

Arnold: “The whole thing.”

Teacher: “Equal parts of the whole? If this names parts of the whole (Pointing to the numerator.) what specific things does this the denominator tell you?”

Student: “The entire whole.”

Teacher: “It’s the parts of the whole but it would be the size? Correct? Remember when we talked about the size represented in a fraction. So, eighth sized piece: an eighth size piece of cake, would that be larger or smaller than a fifth sized piece?”

Arnold: “Smaller.”

Teacher: “So the denominator also talks about size” (as she writes the word size next to the word denominator in parentheses.)

Deborah continues her lesson by directing the students to the book. First, she asks the students to look at the picture in the book of a loaf of bread. She reads the word problem: “Miss Clark made a loaf of bread, and she used one eighth of the bread for a snack and five eighths for lunch. How much of the bread did she use for a snack and lunch?” (Dixon, Leiva, Larson, & Adams, 2012, p. 275).

After she reads she asks the students, “So one eighth, is a one eighth sized piece, is that correct? Can you shade in one eighth in that model to represent the bread? We’re in this picture here (Pointing to where the representation is in the book). So, we’re doing
the whole thing. (She notices Ted has not started coloring the model with the first eighth.)
Can you start over here Ted, because this measurement shows what she used for a
snack?” (Another student is not where they are supposed to be. She points out where they
need to be.) “Color in there one eighth to show what she ate for a snack. And then it says
that five eighths she used for her lunch. So how many eighth sized pieces are you going
to color in to show five eighths?” A student answers five eighths. She corrects him and
says five; “You’re going to color in 5 eight-sized pieces to show what she had for lunch.
So those shaded pieces represent what?”

Another student mumbles something. She continues, “Yes, they represent what
she ate. And how many slices is that all together? So, she used 6 eighth-sized pieces of
the bread. So, eight represents the parts of the whole and six the part that she used.”

“Another way is to use your fraction strips,” Deborah states to her students, “So
take out our fraction strips.” Ben has already modeled the size in the problem with his
fraction strips. Deborah asks him to explain why his model has six pieces.

Byron: “Because, one sixth times five sixths equals one sixth. I mean plus. One
sixth plus five sixths…”

Deborah: (Interrupts.) “No, we’re doing eight.”

Byron: “One eighth times five eighths equals…”

Deborah: (Interrupts.) “No, plus we’re adding.”

Byron: “One eighths plus five eighths equals six eighths.”

Deborah supports Byron so he can explain why he has 6 one-eighth pieces on his
fraction strip. But as I watch I cannot be sure if he just misspoke or misunderstood.
Deborah asks the students to explain how the numerator is represented in this model related to the model presented. Several students respond that the numerator is the parts of the loaf of bread. Deborah pushes for a better answer, “Even more than that.” A student pipes in, “She ate.” Then Deborah asks about the denominator in the model. “The whole loaf,” says a student. Deborah adds to the student’s statement, “The pieces the equal sized pieces. The equal sized pieces that she cut her whole loaf into.”

She then asks, “Did she use the whole loaf?” One student answers yeah, and another answers no. Deborah ignores the yes answer and focuses on the student who answered no, while she continues her questioning, “How many eight-sized pieces did she use?” A student answers six.

Deborah praises the student with the correct answer and continues with the lesson moving on to another word problem: “Jacob needs two strips of wood to make the mast on a miniature sail boat. One mast will be three sixths foot-long the other mast will be two sixths foot long. He has a strip of wood that is four sixths foot long. Is this strip of wood long enough to make both masts?” (Dixon et al., 2012, p. 276).

After reading she tells the students to shade in the model to show three sixths plus two sixths, then write their sum for three sixths plus two sixths. As the students work she narrates, “So we’re just adding the numerators together. So is the sum less than or greater than four sixths?”

The students chorus, “Greater.” “So, does he have enough to build the masts?” she continues. Several student popcorn the answer, “No?” She asks again, “Okay, does he have enough wood?” The students continue to say no. Deborah asks, “Okay, we’re
coloring in how many sixths?” and answers, “five sixths. So, he doesn’t have enough because he needs five sixths, but he only has four sixths.”

Deborah reminds the students that they have previously used a number line and used one-half as a benchmark. She asks the students how we can use a number line with zero at one endpoint, one-half in the middle, and one at the other endpoint. She draws a number line on the board and writes the equation 3/6 + 2/6 = 5/6.

Teacher: “How do you know that our sum is greater than four sixths?”
Arnold: “Just the numerator alone equals five. Not five plus one equals four.”
Teacher: “Can you use the half as a benchmark?”
Arnold: “Yes, because three sixths is equal to a half.
Teacher: “Yes, very good. So, if we’re adding three sixths and we’re adding two sixths more…when we move to the right on a number line our numbers are greater. Correct?”
Arnold: “Three sixths is the same as a half.”
Teacher: (Places 4/6 on the number line and 5/6 on the number line.) “And the numbers to the right going further up the number line are greater. Okay, so we can use the number line and use one half as our benchmark. What if each mast was two sixths feet long, could he use the strip of wood to make both masts?”

Some students tentatively answer, “Yes.” Deborah continues, “Okay so how does that work.” She writes as she says, 2/6 + 2/6 = 4/6.” How long is his strip of wood?”
Students answer, “Four sixths.” Deborah continues, “So he would have enough.”

Deborah is carefully following the lesson in the teacher’s edition of the *Go Math* textbook, which is somewhat scripted. She uses the problems and asks most of the
questions posed in the math book. She reads the next problem, as she writes $\frac{3}{5} + \frac{1}{5}$ on the board: “Jacob needs two strips of wood to make the mast on a miniature sail boat. One mast will be three sixths foot-long the other mast will be two sixths foot long. He has a strip of wood that is four sixths foot long. Is this strip of wood long enough to make both masts?” (Dixon et al., 2012, p. 276).

Teacher: “So in your picture you see one. What does one represent?”

Students: “The whole bag.”

Teacher: “And how many one fifths parts would equal the whole bag?”

Student: “Five.”

Teacher: “Five fifths. So, let’s add together, $\frac{3}{5} + \frac{1}{5} = \frac{4}{5}$.” (She narrates as she writes the answer on the board.) “So how many fifth size pieces did he use?”

Student: “Four.”

Teacher: “So he didn’t use the entire bag. He used four fifths of the bag. Okay, turn to page 287. I’d like you to do two and three on your own. You can see one representing the whole and the fraction parts underneath. So, go ahead and do two and three.” (After erasing the board.) “So, we’re adding the fraction parts together, that is our numerator,” (Students mumble, it may have been a question.) “because the sizes aren’t changing. That’s why we aren’t adding the denominators. We’re just adding the numerators. We’re just adding the parts. On [number] two, one fourth plus two fourths equal what?”

Students answer in a group, “Three.” She asks for the answer of the next problem, three tenths plus six tenths. She answers, “Nine tenths,” as the students also chorus the answer, and then directs the students to problem 13: “Jean is putting colored sand in a jar.
She filled two tenths of the jar with blue sand and four tenths of the jar with pink sand. Describe one way to model the part of the jar filled with sand.” (Dixon et al., 2012, p. 277).

Deborah asks the students “What model can you show?” She points out that the book shows this jar, but asks the students to think of another model they could show. A student responds, “I guess, I could show a bar.”

Deborah tells Arnold, “Okay, for number 10, Arnold, show your model.” As she waits for students to come up with a model, she draws on the board, a rectangle divided in half horizontally with a one in the center of the top rectangle on the board; the bottom rectangle is divided vertically into 10 equal rectangles.

After some wait-time she continues. “Okay, she has two tenths with blue,” Deborah says as she colors two sections of the bottom rectangle blue. She reaches for a red marker and colors the next four rectangle sections, as she adds, “And four tenths with pink.”

“So how much of the jar did she use altogether?” Deborah asks. A student answers “Six tenths.” Deborah writes \( \frac{2}{10} + \frac{4}{10} = \frac{6}{10} \) as she says it.

Next, she moves the student’s attention to the Higher Order Thinking problem. She admonishes the students not to shout out the answer if they know it. She reads the problem, “A sum has five addends and each sum is a unit fraction. Remember what is a unit fraction?” The student’s answer, “A fraction that has the numerator of one.” She continues reading, “And the sum is 1. It has five and each fraction is a unit fraction. So we know that the numerator is one. Correct.” She writes five ones on the board. \( 1 + 1 + 1 + 1 + 1 = 1 \). The students work individually. When they have found and answer she

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writes fives as the denominator of all the ones on the board and asks, “So what is your sum? Some students answer, “Five fifths.” She continues, “Five fifths equals?” as she writes it on the board. Again, the group answers, “One.”

“Right,” she tells them, “That’s a fraction name for one. Remember that’s a fraction name for one. And remember we use the fraction name for one to multiply and divide to make equivalent fractions.” She asks the students if anyone has any questions and gives them the assignment. The lesson is over.

Small Group Instruction – Group Two

Deborah calls the next group. This lesson starts with a warm exchange between the teacher and the students about their scores on a previously given test. Students ask Deborah what their scores were for the previous unit test. She provides scores and congratulates them on how well they are doing.

The students are about to begin a new unit. The Go Math textbook begins each unit with a review of what you need to know to be successful in the unit. This preface to each unit allows teachers to assess student prior knowledge before starting the unit. This is what the group is working on during this lesson.

The previous unit covered simplifying fractions, ordering fractions, and comparing fractions. Now the students are asked to name the fraction for the shaded part and the non-shaded part of a picture. As Deborah explains what the students are supposed to do, there is a cry of confusion from one student. Another student says, “Well, you can’t put eight and four. It’s going to have to be eight, four, and [unintelligible]”.

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Jane is writing whole numbers. Deborah leans toward Jane and reminds her that what she is supposed to be writing is a fraction and that fractions are supposed to have denominators and numerators. Then Jane says, “Yeah, I did.”

Deborah probes further, “No, what does the shaded part represent?” Jane explains that the shaded part represents the numerator, no longer sure of herself.

Teacher: “How many parts represent the whole?”

Student: “Eight? No twelve.”

Teacher: “Yes, there you go. (As she turns away.)”

Student: “And what is the un-shaded part? (Gasp) Oh eight.”

Deborah comforts the students’ confusion by telling them that they are just beginning fractions and confusion is justified. A student responds, “Just a little bit?” He is likely thinking of the unit they just completed.

The students seem to have completed the unit pre-activity. Deborah asks them to identify the characteristics of fractions equal to one. A student defines a fraction that is equal to one as a fraction where the numerator and denominator are the same. Deborah agrees with the student and writes $\frac{4}{4} = 1$ on the board. She moves on to the next problems. The students are given a bar divided into eight equal parts with one part shaded. Deborah asks the student to identify how many parts make up the whole. The student’s response is one eighth, which is incorrect. She asks the question again, “What part of the fraction is that?” Another student answers, seven eighths, which isn’t what she’s looking for either. She repeats the question again, writes a fraction with a question mark as the numerator and eight as the denominator on the board, and asks “What do you call that?” Now the students all answer with strong confident voices, “The denominator.”
The students can identify the numerator as one. Still, one student is confused saying “What!” Deborah continues describing the numerator as the part that is shaded and the denominator is the equal parts of the whole. She writes, “Equal sized parts,” next to the 8 then boxes the word size. As she emphasizes, “So when you’re talking about the denominator you’re talking about the size of the parts.” With the example of the first problem the students breeze through the next few problems.

Given four twelfths, the students correctly identify that it is not in its simplest form. Deborah asks the students, “How do you know four twelfths is not in its simplest form?” Several hands immediately go up. She calls on Jane.

Jane: “Because you could divide four twelfths …divide four twelfths into four fourths equals one third.

Deborah writes Jane’s equation on the board $\frac{4}{12} \div \frac{4}{4} = \frac{1}{3}$ and asks is $\frac{1}{3}$ in its simplest form? The students chorus “Yes.” Deborah continues, “And how do we know it is?” and calls on Gary.

Gary: “Because it’s only common, (pauses) its only common factor is one and itself.”

Deborah: “Okay the only factor is one. Very good! (Turning back to Jane.) Why did you choose four fourths to divide four twelfths by?

Jane: “Because I figured…I figured that that was the biggest…that’s the number that could divide it. Because if you just do one over one, wait no one-half?

Student: “No, one-half is equal to one.”

Gary: (Interrupts) “You can’t divide it by one over one because it’s going to equal itself.”
Jane agrees with Gary. Deborah presses Jane again, “But why did you choose four? Jane, “Because that’s the biggest number that you could divide by.” Deborah asks the question again and picks another student to answer, “Because that’s a common multiple for both the numerator and the denominator. Deborah corrects him, “It’s a common factor.”

Deborah: “Now somebody said you could divide it by two-halves. But then what happens?” (As she writes $4/12 ÷ 2/2 = 2/6$.)

Gary: (Interrupting) “But that’s not in its simplest form.”

Deborah: “So then what would we do?”

Class: “Divide by 2/2 again.” (Deborah writes $2/6 ÷ 2/2 = 1/3$.)

Deborah: “So if you started with 2/2 you would get your fraction, but then you would have to divide again. So, when Jane said it’s the largest number you were choosing the largest factor.”

Deborah ends the lesson by showing students that if you add the shaded part of a fraction and the un-shaded part of a fraction it equals 1. The students are aware that it’s time for lunch. You can hear one say tick-tock, implying it’s time to move on.

Overall, the Richness of the Mathematics in this lesson did not exemplify high quality use of the strategies of this dimension. In this lesson Explanations were more procedural than conceptual. There was little done or said to enhance student sense making. Additionally, the class considered few patterns of generalizations during the lesson. A summary analysis for Observation Seven can be seen in Table 12.
### MQI Analysis for Observation Seven - Grade Four

<table>
<thead>
<tr>
<th>MQI Dimension</th>
<th>Segments</th>
<th>Scores</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>RICHNESS OF THE MATHEMATICS</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Linking Between Representations</td>
<td>S1</td>
<td>N</td>
<td>In the first segment, students listened passively while the teacher</td>
</tr>
<tr>
<td></td>
<td>S2</td>
<td>L</td>
<td>read the answers to the previous assignment without discussion.</td>
</tr>
<tr>
<td></td>
<td>S3</td>
<td>H</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S4</td>
<td>H</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S5</td>
<td>N</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S6</td>
<td>N</td>
<td></td>
</tr>
<tr>
<td>Explanations</td>
<td>S1</td>
<td>L</td>
<td>Explanations were procedural; in some instances, clear conceptual</td>
</tr>
<tr>
<td></td>
<td>S2</td>
<td>L</td>
<td>explanations were not strongly seen in these segments.</td>
</tr>
<tr>
<td></td>
<td>S3</td>
<td>M</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S4</td>
<td>L</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S5</td>
<td>M</td>
<td></td>
</tr>
<tr>
<td>Mathematical Sense-Making</td>
<td>S1</td>
<td>L</td>
<td>Some explanations in this segment were procedural and did not provide</td>
</tr>
<tr>
<td></td>
<td>S2</td>
<td>L</td>
<td>for deep understanding of mathematical concepts.</td>
</tr>
<tr>
<td></td>
<td>S3</td>
<td>H</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S4</td>
<td>L</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S5</td>
<td>M</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S6</td>
<td>M</td>
<td></td>
</tr>
<tr>
<td>Multiple Procedures or Solutions</td>
<td>S1</td>
<td>N</td>
<td>There was lonely minor of multiple procedures or solutions.</td>
</tr>
<tr>
<td>Methods</td>
<td>S2</td>
<td>N</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S3</td>
<td>M</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S4</td>
<td>N</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S5</td>
<td>N</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S6</td>
<td>L</td>
<td></td>
</tr>
<tr>
<td>Patterns and Generalizations</td>
<td>S1 – S3</td>
<td>L</td>
<td>Class did not consider multiple examples and draw conclusions from</td>
</tr>
<tr>
<td></td>
<td>S4</td>
<td>M</td>
<td>them. There was some discussion about generalizations, but it was not</td>
</tr>
<tr>
<td></td>
<td>S5</td>
<td>M</td>
<td>the main part of the segment.</td>
</tr>
<tr>
<td></td>
<td>S6</td>
<td>L</td>
<td></td>
</tr>
<tr>
<td>Mathematical Language</td>
<td>S1</td>
<td>L</td>
<td>The teacher used mathematical language as a vehicle to convey content.</td>
</tr>
<tr>
<td></td>
<td>S2 – S4</td>
<td>M</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S5</td>
<td>H</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S6</td>
<td>H</td>
<td></td>
</tr>
</tbody>
</table>
### Table 12—Continued

<table>
<thead>
<tr>
<th>MQI Dimension</th>
<th>Segments</th>
<th>Scores</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>WORKING WITH STUDENTS AND MATHEMATICS</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Remediation of Student Errors and Difficulties</td>
<td>S1</td>
<td>L</td>
<td>Teacher provided procedural support for student errors.</td>
</tr>
<tr>
<td></td>
<td>S2</td>
<td>L</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S3</td>
<td>N</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S4</td>
<td>N</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S5</td>
<td>N</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S6</td>
<td>L</td>
<td></td>
</tr>
<tr>
<td>Teacher Uses Student Mathematical Contributions</td>
<td>S1 – S5</td>
<td>L</td>
<td>Teacher did not use student comments to move understanding forward.</td>
</tr>
<tr>
<td></td>
<td>S6</td>
<td>M</td>
<td></td>
</tr>
<tr>
<td>COMMON CORE-ALIGNED STUDENT PRACTICES</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Student Provide Explanations</td>
<td>S1 – S4</td>
<td>N</td>
<td>Students did not provide explanations, only pro-forma answers to procedural questions.</td>
</tr>
<tr>
<td></td>
<td>S5</td>
<td>L</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S6</td>
<td>L</td>
<td></td>
</tr>
<tr>
<td>Student Mathematical Questioning and Reasoning</td>
<td>S1 – S6</td>
<td>N</td>
<td>Student comments did not show evidence of student questioning or reasoning.</td>
</tr>
<tr>
<td>Student Communicate about the Mathematics of the Segment</td>
<td>S1 – S3</td>
<td>L</td>
<td>There was minimal student communication in these segments.</td>
</tr>
<tr>
<td></td>
<td>S4</td>
<td>M</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S5</td>
<td>L</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S6</td>
<td>M</td>
<td></td>
</tr>
<tr>
<td>Task Cognitive Demand</td>
<td>S1</td>
<td>N</td>
<td>Students showed evidence of engagement through emotional reactions and statements.</td>
</tr>
<tr>
<td></td>
<td>S2</td>
<td>L</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S3</td>
<td>L</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S4 - S6</td>
<td>M</td>
<td></td>
</tr>
<tr>
<td>Student Work with Contextualized Problems</td>
<td>S1</td>
<td>N</td>
<td>Most of the math work was contextualized with word problems.</td>
</tr>
<tr>
<td></td>
<td>S2 – S6</td>
<td>L</td>
<td></td>
</tr>
</tbody>
</table>

**Observation Eight - Grade Four**

This group of students are starting unit six in *Go Math* fourth-grade textbook. The teacher is working with four students while the rest of the class is working on their math
rotation activities. The lesson started with a vocabulary review of the definitions of numerator, denominator and fraction name of one. The students are listening closely and nod with understanding as Deborah explains the meanings of these words.

After the vocabulary review, she focuses student attention to their textbook and reads a word problem: “Joe took a pan of brownies, and he cut them into third sized pieces. He kept one third and gave the rest away. He’s not going to eat all his part all at once. How can he cut his part into smaller equal sized pieces?” (Dixon et al., 2012, p. 227). The students follow along in their books.

She turns to the board and draws a rectangle to represent the pan of brownies with two vertical lines dividing the rectangle into thirds. Then she asks the students, “What part of the entire piece of brownie did he keep for himself?”

“One third,” Eden answers. Deborah rephrases Eden’s answer as she shades in one third. “So, he kept one of the pieces out of the three pieces.”

Looking back at the word-problem, Deborah tells the students that they are still talking about the same size piece, but Joe will divide it into small parts, as she divides the shaded area in half. Next, she asks the students to imagine what if Joe divided each third in half and adds the lines on the board. “Now how many pieces is the pan divided into?” she asks. Nikita answers, “Six.” Deborah writes /6 on the board.

“Remember, six tells you how many parts there are out of the whole pan.” She continues her explanations, “He has six. How many parts are shaded?” A student answers, “Two.” Deborah adds the two to make two sixths as the fraction on the board.
“So, two-sixths is our new fraction. So, you can rename one third is the same as two sixth.” Although the students seem attentive, there seems to be a tentative air. Two students look at Deborah; the other two are earnestly looking at their math textbooks.

She moves on, “Now look at part B of the problem. Suppose that Joe cuts his part of the brownie into four equal pieces.” She turns to the board and continues, “So this part here we’re going to cut it into four equal pieces.” She carefully divides the representation on the board and the students follow suit in their books. “So now he has four equal sized pieces out of the whole pan of brownies. Now suppose he decides not only in his part of the brownie, he is going to cut the whole pan just as he cut his own. Now how many pieces did he cut his pan of brownies into?” Evan, says “Twelve.” “Now he has Twelve,” she says, as she writes /12, “And how many parts are shaded in?” Two students confidently answer, “Four.” “So, this part, the numerator, is how many parts? We’re counting four,” as she writes and draws at the fraction four twelfths on the board.

Deborah says, “We have one third, which is equal to two sixths, which is equal to four twelfths.” as she writes 1/3 = 2/6 = 4/12 on the board. Again, the students’ body language is passive—they don’t seem to understand. Nikita’s head is cocked to one side and her shoulders are slumped.

Deborah says, “Do you notice that this numerator is doubled?” All four students’ affect changes with this statement. Eden answers with a smile, “Yes.” As an observer, I’m not sure these children understand what she means by equivalent. Since the students are only answering procedural questions, it’s hard to tell what they really understand.

Instruction continues and Deborah makes a mistake in explaining how to find equivalent fractions. “Now we see that two is two times more than four, and three is two
times more than six. We see that we can multiply the original fraction by a fraction name for one to make equivalent fractions,” says Deborah with an air of satisfaction. Deborah sounds certain, case closed, the timber of her voice announces that she is satisfied with her proof. The students still look puzzled. One student, Eden, raises her hand. Deborah continues. Eden lowers her hand.

“If two sixths is twice as much as one third, what fraction name should we multiply one third by?” queries Deborah. Igor answers, “Two halves.” “That’s right,” Deborah says. Now she wants the students to see that they can use number common multiples to find equivalent fractions. She continues, “So let’s multiply,” pointing to one third times two over two, “One times two equals two, and three times two equals six. Look at your model. You can see when you divided it you generated a fraction of two sixths. Now notice that two sixths is twice for four twelfths. Two times more than four. So, what can we multiply by, what fraction name of one do we multiply to get four twelfths?” Students reply, “Two halves.”

Deborah says, “Now go back to when Joe cut his brownie into thirds. The numerator four is four times more than one, and twelve is four times more than three. So, what fraction name of one can we multiply one third to get four twelfths?” Students chorus, “Three thirds.” Deborah corrects them by asking let’s try again? Students answer, “four fourths” Deborah agrees with them and writes the equations on the board, $1/3 \times 4/4 = \_$. She then asks the students $1 \times 4 = 4$ and $3 \times 4 = 12$ and writes $4/12$. “So, you can see how those fractions are equivalent,” Deborah concludes.

Satisfied that the students are following her explanations, Deborah now has students make comparisons using fraction strips. In this next part of the lesson, students
will be comparing fractions. The first fractions they will compare are two fourths and three eighths. She instructs the students to take out their fourths and eighths. The students lay out the strips one above the other. Deborah warns the students to line up the strips so they can compare them. She also helps a student by cutting one of his fraction strips more carefully. Deborah asks if the two fractions are equivalent. Eden answers, “No.” Now, Deborah writes on the board $\frac{2}{4} \neq \frac{3}{8}$. Deborah points out, “You can see using your fraction strips that two fourths is not equivalent to three eighths, and she places $\neq$ between the fractions.

Deborah then tells the students that you can use the fraction strips or multiply by a fraction name for one. She writes $\frac{2}{4} \times \frac{2}{2} = \frac{4}{8}$, asks the students to multiply across. $2 \times 2 = 4$ and $4 \times 2 = 8$, she adds to the equation $\frac{2}{4} \times \frac{2}{2} = \frac{4}{8}$. Next, she has the students add another eighth to their fraction strips and asks if they’re equal. “So, you can use your fraction strips or you can use multiplication by multiplying by a fraction of four,” she states.

As the lesson moves on, Deborah has the students turn to the next page. On this page, each problem has two representations using fraction strips. She proceeds to solve the first problem on the page. She draws a rectangle divided into five parts and shades one part, then she asks the students what fraction is represented by the shaded part. Eden responds, “One fifth.” Deborah draws a second model and asks what we can do to the second model to make equivalent fractions. One student responds, and Deborah restates, “You can divide the pan of brownies in half.” Another student nods her head vigorously in agreement. Deborah asks, “Do we still have the same shaded part? Students answer no. “How many shaded parts do we have now?” She continues, “So ten is our denominator,
and how many parts are we counting?” The students chorus, “Two,” as Deborah writes it on the board $1/5 = 2/10$. “You can draw a model, if this helps you to see it,” she tells the students, “Or you can use a fraction name for one. Let’s try that method.” She writes $1/5 \times \frac{?}{?} = \frac{?}{10}$. “What fraction name for one do you have to use to get this denominator to equal ten?” she asks. The students answer half. She adds to her equation $1/5 \times 2/2 = 2/10$.

“So, two-tenths is equivalent to $1/5$. It still names the same parts,” she states, and moves on to the next problem.

She asks for a volunteer to explain how the next problem should be solved. As the student explains she draws a rectangle divided into three equal sections then shades two sections. The student identifies this fraction as two-thirds, as Deborah writes two thirds next to the representation. Next Eden says, “You divide the shape into thirds vertically.”

Eden continues, “Now there are nine parts with six shaded.” Deborah writes six ninths, then asks, “Is the same area still shaded?” All the students nod in agreement. “So, if we were to multiply this fraction to get a denominator of nine what fraction name for one would we use?” Igor answers as Deborah writes, “Three thirds.” They multiply two thirds times three thirds equals six ninths. Deborah tells them those are the parts that we are counting.

Deborah has the students try problems five and six independently. While the students work, she tells them they will solve the problems mathematically after they’re done. As they finish, each student shows her their book.

Deborah asks Evan to explain how he solved the first problem. Evan explains by writing and saying, “Five eighths comparing that to two fourths.” Victor says, “It is not equal.” Deborah confirms his thoughts and asks him to explain why. (The video doesn’t
allow me to hear Victor’s response.) Victor states, “Five eighths is not equal to two over two because you can’t multiply anything by two to get to five. The denominator eight divided by two is four.” (Evan adds something unintelligible on the video.)

Teacher: (Deborah does not address Victor’s imprecise statement and re-states what he said.) “So, your numerator and denominator, you have to be able to multiply them by a fraction name for one. So, your numerator and denominator must be the same. So, Evan says, if you look, four times two is eight, but you can’t multiply anything by two to get five. Because five is not a (leaving the statement incomplete for students to finish).”

Several students respond with a variety of mathematical terms, “Even number,” “Is an odd number,” “Is not a factor of two.” Finally, she gives them a clue, “Not a mm—.” Then they all say, “Not a multiple of two.

She re-voices their statement, emphasizing the word multiple and discusses the procedure further. “Not a multiple of two. There’s nothing that you can multiply two by and get five, because five is not a multiple of two. Now, four is a multiple, I mean eight is a multiple of four. But remember you must be able to multiply by a fraction name for one. I also heard you say the word divide, so remember the inverse of multiplication is division. So you could have gone this way five divided by two. Oh, that won’t work. But eight divided by two is four. But since you know that five is not a multiple of two, immediately you would know they’re not equal.”

She stops to ask the students which problem they are working on next, then writes the problem on the board, “We’re comparing 4/12 to 1/3. Would somebody like to walk
us through that one?” Matthew: “Four is a multiple of one. (The student pauses. Deborah restates.) And four times three equals twelve.”

Deborah completes his thought, “So they are equal. Do you all agree?” The students answer by nodding. She continues, “So if I were to multiply one third, times what fraction name for one would I have to use to get four twelfths?” A student answers, “four fourths.”

Deborah provides a recap of the lesson and reminds the students that they can use models, fraction strips, or fraction names for one to figure out the answer. She gives them homework and tells them they did a good job. The lesson is over and students return to their places.

Deborah provided students with strong explanations, multiples solution methods, and used mathematical language during the lesson. Students’ communication was mostly procedural. For the most part, the students did not work on contextualized problems, nor did the tasks require a high degree of cognitive demand. A detailed summary analysis for Observation Eight can be seen in Error! Reference source not found.

**Observation Nine - Grade Five**

Today’s lesson starts with a quick review of the math homework the students completed yesterday. Rosita asked the students to carefully review the homework they completed the prior day. The students were instructed to look for areas they found difficult. She posted the homework on the board. She reminds students that they are learning how to divide numbers with zeros in the quotient. She also discusses the homework options they will have today.
Table 13

MQI Analysis for Observation Eight - Grade Four

<table>
<thead>
<tr>
<th>MQI Dimensions</th>
<th>Segments</th>
<th>Scores</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>RICHNESS OF THE MATHEMATICS</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Linking Between Representations</td>
<td>S1</td>
<td>N</td>
<td>Fractions were represented using models, drawing and digits. Linkages between representations pointed out.</td>
</tr>
<tr>
<td></td>
<td>S2</td>
<td>M</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S3</td>
<td>H</td>
<td></td>
</tr>
<tr>
<td>Explanations</td>
<td>S1 – S3</td>
<td>H</td>
<td>Teacher provided step-by-step explanations.</td>
</tr>
<tr>
<td>Mathematical Sense-Making</td>
<td>S1 – S3</td>
<td>M</td>
<td>Teacher provided opportunities for students to make sense of math.</td>
</tr>
<tr>
<td>Multiple Procedures or Solutions Methods</td>
<td>S1</td>
<td>N</td>
<td>Teacher showed students multiple procedures for solving problems.</td>
</tr>
<tr>
<td></td>
<td>S2</td>
<td>H</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S3</td>
<td>H</td>
<td></td>
</tr>
<tr>
<td>Patterns and Generalizations</td>
<td>S1</td>
<td>L</td>
<td>Teacher pointed out patterns in fractions meaning</td>
</tr>
<tr>
<td></td>
<td>S2</td>
<td>M</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S3</td>
<td>M</td>
<td></td>
</tr>
<tr>
<td>Mathematical Language</td>
<td>S1 – S3</td>
<td>H</td>
<td>Teacher and student conversation was dense with mathematical language.</td>
</tr>
</tbody>
</table>

| **WORKING WITH STUDENTS AND MATHEMATICS** |          |        |                                                                      |
| Remediation of Student Errors and Difficulties | S1 – S3  | N      | Students did not make errors or asked for help understanding material. |
| Teacher Uses Student Mathematical Contributions | S1       | L      | Only the last segment contained student communication aside from pro-forma answers. |
|                                | S2       | N      |                                                                      |
|                                | S3       | L      |                                                                      |

| **COMMON CORE-ALIGNED STUDENT PRACTICES** |          |        |                                                                      |
| Student Provide Explanations       | S1       | N      | Students provided explanations in the last segment. Teacher was talking most of the time. |
|                                | S2       | N      |                                                                      |
|                                | S3       | M      |                                                                      |
| Student Communicate about the Mathematics of the Segment | S1     | L      | Only the last segment contained student communication aside from pro-forma answers. |
|                                | S2       | N      |                                                                      |
|                                | S3       | H      |                                                                      |
Table 13—Continued

<table>
<thead>
<tr>
<th>MQI Dimension</th>
<th>Segments</th>
<th>Scores</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Task Cognitive Demand</td>
<td>S1</td>
<td>N</td>
<td>Students were not disruptive, but their body language seemed to hint that they were not engaged in making meaning of the math.</td>
</tr>
<tr>
<td></td>
<td>S2</td>
<td>N</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S3</td>
<td>M</td>
<td></td>
</tr>
<tr>
<td>Student Work with Contextualized Problems</td>
<td>S1</td>
<td>N</td>
<td>There was a word problem for the first segment only.</td>
</tr>
<tr>
<td></td>
<td>S2</td>
<td>N</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S3</td>
<td>L</td>
<td></td>
</tr>
</tbody>
</table>

Rosita asks the class to name a problem from the problems they had completed the previous night for homework. The students decide to use problem number four, 30 divided by 0.8. Rosita asks Gavin to come to the board and work out the problem.

Gavin says okay, hesitantly. Rosita reassures him that the class will help him out if he gets stuck. Gavin comes to the board and begins to work out the problem. As he starts working on the problem Rosita narrates as he writes.

Rosita: “First thing we do is to move the decimal point.”

Gavin begins writing on the board, he places the decimal to the right of eight in the dividend. His hand hovers over the three and zero of the divisor. He settles by putting the decimal point next to the zero in the divisor.

Rosita: “Because if you don’t have a decimal in the divisor what do you assume?”

Unidentified Student: “It’s at the end.”

Rosita: “Right, it’s at the end.”

Gavin now starts to use the division algorithm to divide the number. He places a three over the three in the dividend and writes 24 below the 30. Using the subtraction algorithm, he subtracts $30 - 24 = 6$. Writing in the small space between the divisor and
the long division symbol is difficult with a marker. He places a dot and what seems to be a one over the three. This seems to represent ten while he is subtracting. Rosita affirms his action then asks what he should do next. Gavin says bring down the zero and proceeds to do it. Rosita stops him and asks what he should do to the top of the algorithm, [meaning the quotient]. Gavin adds a decimal point next to the three. She tells him that he needs to add a placeholder because you can’t divide six by eight. Gavin adds a zero after the decimal point. Gavin is lost; he doesn’t know what to do next. Rosita tells him he can get help from a classmate. He calls on an unidentified student who tells Gavin to use seven. With a smile, Gavin turns back to the board and writes the seven after the zero. He writes a 56 under the 60 and uses the subtraction algorithm to get four as the answer. Rosita reminds him that you can’t have a remainder, and then she asks him what he needs to add. Gavin adds a zero after the four to make 40. He asks if he needs to add a zero to the quotient. Rosita tells him it is not needed. Gavin places a five after the seven. As he starts to write 40 under the 40, Rosita states, “I think we made a mistake. Where did we make the mistake?”

Several students chorus, “Zero.”

Rosita: “This is where we made the mistake. I told him to put the zero up there and I was wrong. So, it should be three point seven-five Yeah, okay.” She confirms her own thinking.

Unidentified Student: “I got 37. 5.” [Several students agree with this student.]

Rosita: “So how many of you got 3.75? How many of you got 37.5” [emphasizing the 37]?
Mary: “Well that would be higher. It can’t be that way. It would be higher than 30.”

Rosita: “Aha! If we had it as 375 it would be higher. But is 37 higher than 300? Gavin, please move the decimal point. Because what did we forget to do after we added the zero? What did we forget to do when we added the decimal? When we moved it [the decimal point] on the eight, but we didn’t move it [the decimal point] on the [number on the] right [side].” Rosita walks up to the board adds the corrected decimal point. And Gavin sits down.

Rosita moves on to the next problem. Sheila comes up. She rewrites the expression 138.4/16 into the long division format.

Rosita: “Do you have any decimals in your divisor? Sixteen is a whole number already so you don’t have to move your decimal. So, do you have to move your decimal?”

Unidentified Student: “No.”

Rosita: “Do you want to put the decimal up on top?” [Meaning above the long division symbol.]

Sandy put the decimal point above the one in the problem. Then she starts looking for a number to divide by. She tries 16 x 7. She uses the multiplication algorithm and gets an answer of 112. She decides that 112 is not large enough so she writes 16 x 8. She decides that 128 is her best choice. She goes over to the division problem and writes the 8 above the 8 in division problem. Then she writes 128 below the 138 and subtracts, 138 – 128 = 10. She continues with the division algorithm dividing 104 by 16. On the side, she writes the algorithm for 16 x 6 and works it out to 96. Rosita asks her why she chooses to
use six. Sheila responds that the seven is too big. She continues to work on the problem. She writes six after the decimal point above the problem, she writes 96 below the 104, and then subtracts 104 – 96 = 80.

Unidentified Student: “You forgot to subtract the one on the top.” [Student was referring to the ones in the hundreds place.]

Off Camera Student: “She doesn’t need that number.”

Sheila writes the zero then erases it when her work looks confusing. Rosita commends the students for questioning each other’s work. Sheila finishes by multiplying 16 x 5 = 80 and writing five above the division problem and subtracting 80 from 80.

Rosita: “Voila! Did you all get 8 points 65?”

Sheila erases her work and sits down. Rosita moves on to a word problem and reads it to the class, “Josh pays $7.59 for 2.2 pounds of ground turkey. What is the price per pound of the ground turkey?” She calls Bart up to solve the problem.

Rosita: “What do you think he is going to have to do to solve the problem? What operation will he need to do?”

Class: “Divide.”

Rosita: “Divide, because you’re trying to find price per pound. Okay.”

Bart writes the 759 divided by 2.2 in long division format and starts to move the decimal point to the right of the divisor. Rosita reminds him that he didn’t write the decimal in the dividend when he copied the problem. Bart adds the decimal for the dividend.

Rosita: “Do you have to move it?”

Bart: “Yes.”
Rosita: “How many spaces?”

Bart: “One.”

Rosita: “So now I’m dividing 75 by 22.”

Bart moves off to the side of his division problem and writes $22 \times 3$. He stops, looks around, then writes 5, then he erases the 5. Rosita asks, “What is three times two?” Bart decides to use four. Rosita tells him to try it, but sounds skeptical. Bart gets 88. Rosita tells him he should go back to his original number. Bart gets 66 and writes that number under the 75 in the problem. He writes the three above the five and subtracts $75 - 66 = 9$. He brings down the nine and multiplies 22 by five. He sees 110 is too big then tries four. He writes the four above the zero and writes 88 below the 99. He brings down a zero, and then multiplies five times 22. He gets 110 and finishes the problem. Rosita asks the class if they got the same answer then she moves on to further review the procedure.

Rosita: “Yesterday, when we were doing our division we played a little game. Last night Lindsey made some number cards for us to use. Thank you, Lindsey, we appreciate your hard work. So, I’m going to give you a problem and you will set up the problem. We are going to use the stool as the divisor line. We’re going to ask Maria to help us with the numbers and we are going to see how quickly we can do this today. Our goal today is to learn or to remember how to move the divisor. If you do it to the divisor…decimal, I’m sorry. If you do it to the decimal on the divisor you also must move it in the dividend and you put it up in the quotient. So, your first problem we are going to divide zero point six. Set it up quick.” [Three students get up one holding each
number and the decimal.] “And you’re going to divide decimal two four. What’s the first thing we have to do?”

Rosita: “Decimal where would you move to?” [The student with the decimal in the divisor moves to the right.] “Perfect. Because the other decimal moved what do have to do to this decimal?” [Pointing to the decimal in the dividend.] “Now we have to solve the problem.”

Once the students solve the problem Rosita gives them another problem to solve in the same manner. They are practicing for a competition between the classrooms next week. They repeat the process with three other sets of numbers. Although the students enjoy the activity, the activity does not connect the meaning of their actions with the procedures. They solve several problems this way, and then Rosita thanks the students for their participation, and the class is over.

This lesson focused on using a single algorithm to solve divisions of decimals by whole numbers. The teacher spent considerable amount of time reviewing the algorithmic procedures. There were few student errors, which the teacher addresses procedurally. Although consistent, student communication did not include conceptual insight, mathematical questioning or mathematical reasoning. The tasks did not seem to engage the student and were not placed in a real-life context. A detailed summary analysis for Observation Nine can be seen in Table 14.Observation Ten - Grade Five

This observation occurs at the end of May. The students are courteous and working hard even though it is the end of the school year. Rosita reminds the students that I am there to tape her. Then she begins the class by reviewing what they have been
**Observation Ten - Grade Five**

This observation occurs at the end of May. The students are courteous and working hard even though it is the end of the school year. Rosita reminds the students that I am there to tape her. Then she begins the class by reviewing what they have been

Table 14

**MQI Analysis for Observation Nine - Grade Five**

<table>
<thead>
<tr>
<th>MQI Dimensions</th>
<th>Segments</th>
<th>Scores</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>RICHNESS OF THE MATHEMATICS</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Linking Between Representations</td>
<td>S1 – S3</td>
<td>N</td>
<td>Teacher only used algorithm to solve problems.</td>
</tr>
<tr>
<td>Explanations</td>
<td>S1 – S3</td>
<td>N</td>
<td>Explanations contained appropriate use of procedures without explaining why.</td>
</tr>
<tr>
<td>Mathematical Sense-Making</td>
<td>S1 – S3</td>
<td>N</td>
<td>Teacher did not make any connections between numbers and their meanings.</td>
</tr>
<tr>
<td>Multiple Procedures or Solutions</td>
<td>S1 – S3</td>
<td>N</td>
<td>Only one solution or procedure method was used.</td>
</tr>
<tr>
<td>Methods</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Patterns and Generalizations</td>
<td>S1 – S3</td>
<td>N</td>
<td>Students did not develop generalizations or notice patterns.</td>
</tr>
<tr>
<td>Mathematical Language</td>
<td>S1</td>
<td>N</td>
<td>Teacher use of mathematical language was not consistent.</td>
</tr>
<tr>
<td></td>
<td>S2</td>
<td>L</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S3</td>
<td>N</td>
<td></td>
</tr>
<tr>
<td><strong>WORKING WITH STUDENTS AND MATHEMATICS</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Remediation of Student Errors and</td>
<td>S1 – S3</td>
<td>L</td>
<td>Teacher provided pro forma support for students encountering difficulties in using division algorithms.</td>
</tr>
<tr>
<td>Difficulties</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Teacher Uses Student Mathematical</td>
<td>S1 – S3</td>
<td>L</td>
<td>Student communication was pro forma - did not showcase student mathematical thinking.</td>
</tr>
<tr>
<td>Contributions</td>
<td></td>
<td></td>
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<tr>
<td><strong>COMMON CORE-ALIGNED STUDENT PRACTICES</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Student Provide Explanations</td>
<td>S1 – S3</td>
<td>N</td>
<td>Students did not provide explanations.</td>
</tr>
<tr>
<td>Student</td>
<td>Mathematical Questioning and Reasoning</td>
<td>S1 – S3</td>
<td>N</td>
</tr>
<tr>
<td>MQI Dimensions</td>
<td>Segments</td>
<td>Scores</td>
<td>Notes</td>
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</tr>
<tr>
<td><strong>Student Communicate about the Mathematics of the Segment</strong></td>
<td>S1 – S3</td>
<td>L</td>
<td>Students spent most of the class period explain the procedure for dividing numbers with decimals in the dividend and divisor without explaining why actions were needed. They worked together and were engaged.</td>
</tr>
</tbody>
</table>

| **Task Cognitive Demand** | S1 – S3 | N | Tasks were not cognitively demanding; they did not require students to understand concepts just memorize procedures. |

| **Student Work with Contextualized Problems** | S1 – S3 | N | Students did not use mathematical meaning to solve the given problems. |

...studying. The students know they are in a geometry chapter and Rosita adds that they will be studying volume in today’s lesson.

She asks the class “What is volume?” A student responds with the formula for finding volume. Rosita explains, “We use grams, ounces, and pounds to find out how much something weighs. We use inches, yards, [and] feet [to measure] how small something is…I gave it away! To measure what?” “Length,” A student answers, “Length.” Rosita continues her explanation, “Volume measures how much you can fill something with.”

Rosita: “I love volume, because. Because that means if I have a Rubber-Mate container I want to know how much I can fit into that container. So, I can get more stuff out of my garage. So, volume measures how much something can hold. Volume is how much something can hold. And our formula, Martha? We figured it in a simple way to find the volume?”
Martha: “We multiply the length times the width by the height.”

Rosita: “Today we are continuing on with that same discussion. Today we are going to be measuring some containers to see if you can figure out their volume. You’re going to use inch cubes to build some shapes and figure out the volume.”

Rosita goes to her computer to pull up the math lesson on the electronic white board. She tells me that they had trouble starting the math this morning. After a minute, the math book opens on the computer and Rosita continues instruction. She opens the electronic version of the fifth-grade *Go Math* Textbook to lesson 11.9. Then she asks the students to open their textbooks to the same lesson. Rosita reads the problem, “Sid is building a rectangular prism using one-inch cubes. What would his base unit of measurement be?” There is mumbling, but no student provides a reasonable answer. So, she changes the question, “What measurement did we give for the cube?” Then several students respond “One-inch cubes."

Rosita: “Your answer today will be given in 1-inch cubes. They have taken the figure apart and they have shown you the different levels. When you have a figure, can you always see the sides and the bottom and the back?” [Several students say no.] “When you look at a figure you only usually see the front faces, the sides and the top. You must [be] able to find the measurement by using [unintelligible]. But today we have actually four levels apart to show you that they have equal number of squares, cubes on all four levels. And that’s what you’re going to be doing when you work on your projects in a few minutes. So, each layer of this rectangular prism is made of how many inch cubes? And that’s what we are going to discover right now. How many inch cubes? Where could I look on these levels to see how many inch cubes are on the levels? Could I look at the
bottom, the middle or the top to get that information?” [Students say something inaudible.] “That’s right the top. Because, can I see all the cubes that are in these sections [pointing to levels two, three, and four]?”

Student: “The top.”

Rosita: “So I’m looking at the top. So, Andrew, how [many] cubes do I see at the top?”

Robert: “Uh-mm, 12.”

Rosita: “Okay, so I have 12 here.” [Writing 12 on the top level of cubes.] “…I am going to assume that I have how many on each of the other levels?”

Several Students: “Twelve!”

Rosita writes 12 on each level; then she writes the answer to that question in the textbook. The next question in the textbook has a chart which Rosita and the students complete. Rosita points out that there is a pattern. The top row you count by ones and the bottom row you count by twelve. Using the pictorial representation Rosita shows the students that the lowest level is composed by four rows with three cubes in each. And then she counts-up to show that each level has the same number of cubes. She points out that four times three times four is the same as the answer on the chart. She compares the 48, which was arrived at by using the chart and the 48, which was arrived at by multiplication, showing that there are two ways to get this answer.

Rosita goes on to the next problem. This next problem is essentially the same problem. The only change is that the base unit used is centimeter cubed. Rosita guides the students’ thinking to decide how cubes are found on each layer. When the students don’t answer, she tells them that they should look at the top layer because you can see all the
cubes on that layer. She asks a student to count the cubes and then begins to solve the problem.

Rosita: “I have 24 on the top layer. And I would have 24 on each of the other layers, correct?” [There is a whimpering, “Yes” from a student.] Rechecking the problem, she reminds them there should be no gaps or overlays. She asks the students to find the volume. Everyone quietly works on their own while Rosita walks around the classroom looking over the students’ shoulders at their work. She encourages students to try and see what they get.

After a while she starts to work the problem out on the board. She figures out how many on the base by multiplying length times width, and then she multiplies the product by 6. As she is doing the work on the board she encourages the students to change their answers if they didn’t get the right answer. She asks a student, “What did you get?”

The student answers, “144.” Rosita responds that his answer is correct. Another student raises his hand and says, “I did it by multiplying 6 x 6 x 4.” Rosita asked the class, “Did any of you get the same thing?” When several students answer yes, Rosita moves on. She doesn’t make any connection between why both solution procedures worked. As she erases the board she reminds the students that today in math there can be more than one way to get the answer, unlike when she was in school when there was only one way. “This teaches you different strategies, which is good. Let’s try the way we’ve been learning which is the length times the width times the height”. She fills in the answer on the board. Then she cautions the class that formula strategies are most useful in this part of the lesson.
Next, she divides the class into groups of four. Two groups will use cubes to make models of the boxes they are given. Two groups will use tape measures to figure out the volume of their given boxes.

While the students are getting organized, Sally comes to ask a question. She has a question about the assignment in the textbook. Once Rosita helps the student with a question, she continues passing out materials. As soon as the students have the supplies, they start to work; they sound excited and seem happy to work together. Rosita walks around helping the students organize themselves. She goes from group to group making sure they are correctly doing the task. One student tells her he has forgotten everything she said on the board. Some groups are done and other groups haven’t even started. Other groups are not doing the hands-on task and are working on the assignment in their textbook. This group has questions about how to complete the textbook assignment. Rosita helps them.

After 10 minutes, she collects the materials and gathers the students’ attention and reminds them that the answers need cm³ (cubic centimeter) after the number. She then asks, “Why do we need to put the little three?” A student answers, “Because your times-three things”. Rosita rephrases her answer. “This three-means cubed, it is like when you’re multiplying 10 times 10 times 10.” (This is not the correct reason for the mathematical notation of a centimeter cubed.)

“So, you can put 18 centimeters to the third power. Or to the exponent of three,” Rosita tells the class. Then she admonishes them, “Don’t forget to label your answers.” Rosita asks the students to exchange materials. The students with the tape measure should switch to blocks and the students with blocks should switch to tape measures or rulers.
She gives an additional instruction, “If you’re done measuring the materials I gave you, you can find other rectangular prisms to measure.” Students ask, “Where can we find other rectangular prisms?”

Rosita gives them a few examples and tells the students to look around. There’s lots of rectangular prisms in the classroom. Some students are playing with the blocks. The video tape stops at this point and I am unable to see the last five minutes of class.

This geometry lesson focused on finding the area of rectangular prisms. The teacher provided students with multiple representations using concrete and pictorial representations while drawing the students’ attention to the similarities of the representations. The teacher demonstrated multiple solution methods. She gave the students a hands-on activity, but students didn’t seem engaged in the math concepts represented by the activity. A detailed summary analysis for Observation Ten can be seen in Table 15.

**Observation Eleven - Grade Seven**

Tony teaches all math classes for middle and high school students at his school. This observation takes place in a seventh-grade class, and is composed of two groups. One group is working on a context problem where they must determine what materials they need to buy to make chocolate chip cookies.

The students in this group are divided into groups of three. First, Tony provides the students with an introduction to their task. He points out the recipe and what is in the pantry, and then asks them to provide a shopping list. Then he reads off a list telling the students what group each student is in. With a little prompting, they begin to work.
### Table 15

**MQI Analysis for Observation Ten - Grade Five**

<table>
<thead>
<tr>
<th>MQI Dimensions</th>
<th>Segments</th>
<th>Scores</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>RICHNESS OF THE MATHEMATICS</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Linking Between Representations</td>
<td>S1</td>
<td>H</td>
<td>Teacher described how adding cubes layer by layer will give you the same number of cubes as using the standard formula.</td>
</tr>
<tr>
<td></td>
<td>S2</td>
<td>H</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S3</td>
<td>N</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S4</td>
<td>N</td>
<td></td>
</tr>
<tr>
<td>Explanations</td>
<td>S1</td>
<td>H</td>
<td>Teacher explained how cubes that fill a tub can be counted to give you the number of cubes a tub can hold.</td>
</tr>
<tr>
<td></td>
<td>S2</td>
<td>L</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S3</td>
<td>N</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S4</td>
<td>N</td>
<td></td>
</tr>
<tr>
<td>Mathematical Sense-Making</td>
<td>S1</td>
<td>M</td>
<td>She encouraged the student to see the meaning of the numbers that describe the volume of a cube.</td>
</tr>
<tr>
<td></td>
<td>S2</td>
<td>M</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S3</td>
<td>N</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S4</td>
<td>N</td>
<td></td>
</tr>
<tr>
<td>Multiple Procedures or Solutions Methods</td>
<td>S1</td>
<td>H</td>
<td>She used two procedures to find the volume of a rectangular prism.</td>
</tr>
<tr>
<td></td>
<td>S2</td>
<td>H</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S3</td>
<td>N</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S4</td>
<td>N</td>
<td></td>
</tr>
<tr>
<td>Patterns and Generalizations</td>
<td>S1 – S4</td>
<td>N</td>
<td>She did not draw any attention to patterns or generalizations.</td>
</tr>
<tr>
<td>Mathematical Language</td>
<td>S1 - S3</td>
<td>M</td>
<td>She moderately used mathematical language to describe her thinking.</td>
</tr>
<tr>
<td></td>
<td>S4</td>
<td>L</td>
<td></td>
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<tr>
<td><strong>WORKING WITH STUDENTS AND MATHEMATICS</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Remediation of Student Errors and Difficulties</td>
<td>S1</td>
<td>N</td>
<td>She addressed student calls for help when she was aware that they need help. The use of closed questions inhibited her ability to determine if students understood the concept.</td>
</tr>
<tr>
<td></td>
<td>S2</td>
<td>N</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S3</td>
<td>L</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S4</td>
<td>L</td>
<td></td>
</tr>
<tr>
<td>Teacher Uses Student Mathematical Contributions</td>
<td>S1 – S4</td>
<td>L</td>
<td>She did not use open-ended questions - student contributions were mostly one or two words.</td>
</tr>
</tbody>
</table>
Table 15—Continued

<table>
<thead>
<tr>
<th>MQI Dimensions</th>
<th>Segments</th>
<th>Scores</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>COMMON CORE ALIGNED STUDENT PRACTICES</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Student Provide</td>
<td>S1 – S4</td>
<td>N</td>
<td>The student did not provide any explanations.</td>
</tr>
<tr>
<td>Explanations</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Student Mathematical</td>
<td>S1 – S4</td>
<td>N</td>
<td>There was no evidence of student reasoning or questioning mathematically.</td>
</tr>
<tr>
<td>Questioning and</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Reasoning</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Student</td>
<td>S1</td>
<td>L</td>
<td>Student communication was the result of short closed questions. During the second half of the class the students were working in small groups and I was not able to distinguish what they were saying.</td>
</tr>
<tr>
<td>Communicate</td>
<td>S2</td>
<td>L</td>
<td></td>
</tr>
<tr>
<td>about the</td>
<td>S3</td>
<td>N</td>
<td></td>
</tr>
<tr>
<td>Mathematics of the</td>
<td>S4</td>
<td>N</td>
<td></td>
</tr>
<tr>
<td>Segment</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Task Cognitive</td>
<td>S1</td>
<td>N</td>
<td>Although the tasks could have given students mathematical ideas to work on the teacher scaffolded the assignment and did all the thinking for the students.</td>
</tr>
<tr>
<td>Demand</td>
<td></td>
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</tr>
</tbody>
</table>

The second group of students is given a cardboard cylinder. They should find the surface area. Tony tells them what they will need. He provides this group with a quick review of what the students are asked to do. Then he models the processes and explains, “What we’re going to do is we’re going to cut it out. Then we will have a rectangle. You know how to find the area of a rectangle?” Students whispers, “Length times width.”

Tony’s attention is focused on a student that doesn’t understand. He repeats the original directions about cutting the cylinder. Now the student seems to understand. Other students are chiming in with other formulas, “Height times width.” “Wait, wait, is it base times height?” “L times W times two.” Tony cuts the cylinder and asks the students how to find the area of the cardboard rectangle. The students continue to give random
formulas. Tony tries to focus them by telling them, “We’re not trying to find the volume we’re just trying to find the surface area of this rectangle. Like if I wanted to color this.”

The students seem to get what Tony is asking for. They all chorus, “Length times width…So finding the area of that rectangle is the same as finding the area of this part of the cylinder. But does that give me the area of the entire cylinder? What’s missing?”

Student: “The tops.”
Tony: “What shape is the top?”
Student: “A circle.”
Tony: “How do you find the area of a circle?”
Student: “By that thingy.”
Student: “Two times $\pi$ times the radius to the second power.”
Student: “The radius.”
Tony: “$\pi r^2$.”

After reviewing the formula Tony tells the students, “If you want to tell the area of a cylinder you have to find the area of this [top] circle and this [bottom] circle and this rectangle [using the cardboard cylinder to model].”

A student hesitantly asks, “So do you want us to find the area and pretend that’s hollow? No, I mean pretend that’s filled?” Tony continues, “I want you to find the surface area. That means the area of this circle, this circle, and the area of the surface around it.” There’s still some confusion on how to find the area of a circle. The students seem unsure about the difference between area and volume. One student says, “You square $\pi$.” Tony repeats the formula, then tells the students that they will need to find the radius of the circle. One student asks if he needs to multiply the radius by two. Tony
reminds the students that a number squared is multiplied by itself. Students are left to complete the activity on their own. “I’ll be back.” Tony tells them. Tony returns five minutes later to answer student questions. Tony says, “Okay, well we know what $\pi$ is. We need to find the radius. How do you find the radius?” Pete responds by pointing, “It’s about here. (Points to the middle of the hollow center of the cylinder.) Tony follows up with, “How can we figure it out more accurately?” Several students, call out “Diameter.” “A compass.” “It’s one and one half inches.” Tony focuses on that answer, “How did you figure that out?” Again, the students say several unintelligible things. Tony counters with, “How are you finding the radius by measuring?” The student answer is not heard. Tony restates the student’s explanation, “Because the radius is one half the diameter?” using a question. This leads the students in the right direction. Several begin to explain the steps of what they need to do. Tony continues, “So in your formula you are going to do $\pi$ times one and one half times one and one half.” After a few moments of listening to the students as they work through the ideas, Tony moves on to another group.

Tony now moves to the other group. They were given the chart seen in Figure 4 at the beginning of the lesson to make the shopping list. He steps to the first group and asks how they are doing. He asks the students, “What are the ingredients in the first column?” They answer the ingredients needed for the recipe. He asks, “What about the second column?” They answer the ingredients in the pantry. “So, let’s look at the first item, flour. How much flour do they need?” The students answer his questions, and Tony leads them to decide whether they need to buy flour. Then he moves on to baking soda. Finally, he asks about butter. The students respond, “We do need butter.” Tony questions them
### Recipe

<table>
<thead>
<tr>
<th>Have in pantry</th>
<th>Shopping list</th>
</tr>
</thead>
<tbody>
<tr>
<td>4 1/2 cups flour</td>
<td>10 cups flour</td>
</tr>
<tr>
<td>7 1/2 tsp. baking soda</td>
<td>7 1/2 tsp. baking soda</td>
</tr>
<tr>
<td>2 cups butter</td>
<td>2/3 cup butter</td>
</tr>
<tr>
<td>3/4 cups brown sugar</td>
<td>4 cups brown sugar</td>
</tr>
<tr>
<td>1/4 cup white sugar</td>
<td>1/4 white sugar</td>
</tr>
<tr>
<td>9 eggs</td>
<td>9 eggs</td>
</tr>
<tr>
<td>4 cups chocolate chips</td>
<td>4 cups chocolate chips</td>
</tr>
<tr>
<td>2/4 cup white sugar</td>
<td>2/4 cup white sugar</td>
</tr>
<tr>
<td>1/4 cup walnuts</td>
<td></td>
</tr>
</tbody>
</table>

*Figure 4. Teacher Board Example.*

...further, “How much butter do you need?” Tony leaves them to discuss what the answer should be, and joins another group.

This group needs help in becoming a group. The students are seated near each other but not facing one another. The group is composed of one boy and two girls. Tony helps them turn to face each other. The boy believes there is a mistake in the given chart. He points out that sugar is listed twice in the chart. Tony asks for everyone’s attention. “Hey guys, Milton made an observation.” Tony explains, “As you look in the pantry you find two containers of white sugar, one has a fourth of a cup of sugar and the other has...
three fourths of a cup.” He returns to the group, he lets Milton know it’s not a typo—it’s important information.

Tony asks the group, “Do you guys know what you’re doing?” Loren answers in grouchy tone, “Nooo!” Tony notices that Milton has almost completed the assignment. He turns to Milton and asks him to explain each of his answers. Then he asks the group to decide whether they agree or not. Then he walks away.

At the next group Tony asks, “Have you guys figured out what you need?” This group explains that they need butter and walnuts. One student starts to explain what is needed. Tony walks away. The students can be heard talking about the problem, “We don’t need baking soda. We need butter.” Tony moves from group to group checking how they are doing, listening to their questions or explanations. He asks the groups to come to a stopping point and come together to discuss what they did.

The first presenter, Nate, seems hesitant to present. Tony reviews his work and tells him, “It’s okay, let’s go.”

This first group has some technical difficulties using Airplay, but they finally get it working. “We need butter two thirds and walnut is one over four and of course we added the sugar one fourth plus two fourths is three fourths. Because it says right here that for sugar we need one over three fourths.

Tony: “You have how much butter?”

Student: “Two thirds.”

Tony: “And you need a total of two cups. So, the recipe calls for two cups so how much do you need to buy? (No student response.) All right why don’t you sit down and work on that. Next group.”
The next group logs on to Airplay. While the next group is setting up, Tony helps the first group. One student says, “I still don’t get it.” Tony tells her, “Mary will help you. She knows what to do.” Tony moves to the group that just presented. Robbie asks, “What am I supposed to do, is it divide, multiply . . . (his voice trails off)” Tony asks Robbie, “If you have two thirds and you need two what’s the difference?” Robbie’s answer is confused, “The difference is that you need to put two, you need to put this one. Umm, basically…” Tony asks, “What does the word difference mean in math?”

Robbie: “The different numbers.”

Tony: (Shaking his head, no. In a very patient tone.) “No, it means subtract. So, you need to take those two numbers and subtract them.”

Robbie has another question, but the next group is ready to start. Tony asks for the class’ attention. Milton is the group’s speaker. “All we needed was one and one third butter and one fourth of walnuts.” Tony asks, “Okay, how did you get that?” Milton continues, “Because one fourth of walnut was all we needed and we didn’t have any. And we needed two cups of butter when you add one third and two thirds that’s one cup.”

Tony follows up with questions. “You mentioned the white sugar. Why don’t we need any whiter sugar?” Milton explains that by adding the two white sugars in the pantry you have enough white sugar. Tony asks the girls in the group if they want to add anything. He also asks the class group if anyone has any questions or comments.

Tony asks the students to attend to him. At this point he gives them a short lesson on adding fractions. He tells them, “There are several ways to solve this problem. But one that I saw this group doing was adding fractions. We’ve talked a lot about fractions recently. We’ve worked on multiplying and dividing [fractions], but today we’re talking
about adding fractions. Now if I have a fraction like three over eight. (As he writes three eighths on the board.) What does that mean?”

Several students give different answers, “It’s a fraction.” “It can be divided.” “A third of eight.” Tony restates this answer, “A third of eight,” and follows it up with, “How can I represent three eighths with a picture?” Again, several students answer, “You draw a circle of eight and then you color it in.” “With a pizza.”

Tony draws a circle and divides it into eight pieces. He stops to discuss a different idea, “Hey, wait, what if I draw something like this.” (He draws a circle then divides it in half then divides the top half into eight parts.) Several students respond, “Yes, but that will look random.” “No, you can’t.”

In a questioning tone, Tony says, “That’s eight pieces?”

Several students respond, “Yeah, but then you have the bottom.” “Because it’s not…” “It’s not equal.” “It’s not equal for everybody.”

Tony continues, “Think about this. What about if it’s my birthday and I tell you I will get the bottom half of the cake and you guys have to share the rest.” Students interject, “It’s not fair.” Tony explains, “It’s not fair because they’re not the same size. So, we can’t do this. An important thing to remember is that we must have thee equal parts.” Then he colors three parts of the circle. “Now we have five eighths. How do we do that?” He quickly draws a circle, divides it into eight, then colors in five parts. “Now what does it mean if I add these two fractions”? A student calls out, “It’s one.” The teacher says, “I didn’t ask what is the answer; I asked what it means.”

Again, the students contribute lots of answers, “It means that you’re taking a group of three eighths and adding it to a group of five eighths.”
Tony takes out some blocks and places three in a pile and five in a pile. He has the students say that they need to join the two piles to add them. “So, what does it mean to add?” Several students answer his question, “To add them together.”

Tony picks on their idea, “To put them together. To combine them, so if I have three-eighths and five-eighths and I add them what am I doing to them?”

Several students, “Combining them.”

Tony continues, “So, if I have three slices of pizza in this pie and five slices of pizza in this pie and I combine them I now have eight pieces. As he colors in the empty five slices of the circle with three eighths colored in. “So that’s eight over eight,” a student says. Then another student says, “Which equals one.”

“Something to keep in mind, when you’re dealing with fractions the top number is how many and the bottom number is how big,” Tony states. “These are three eighths because we cut the pizza into eight slices. So, if we add three eighths plus five eighths we’re not adding the eights because the eight tells us, what?” Again, several students answer, “How many.” “The bigger.”

Tony steps in and answers his own question, “The size. The top number tells us how many. So, when we add two fractions; how do we do that?” Several students say, “You can combine them.”

“So, we said that three eighths plus five eighths equals eight eighths. So, look at the pattern. We have three over eight and five over eight equals eight over eight. In our answer, what do you notice that is similar between these fractions?” A student answers, “They have a common denominator.” Tony rephrases the student’s answer, “The denominator does not change. What do you notice about the numerator?” One student
answers, “The numerators are added together.” Another student asks, “What about if the denominators are different?” Tony responds to that student’s questions, “You can’t add that. We’ll save that for another time.” Then he writes two other fractions on the board, \(\frac{2}{3} + \frac{3}{3} =\), and asks, “How can you add these?” Students again talk over each other, “You could add the numerators and then simplify.” “You could do like a box and cut it into threes.”

Tony asks the student, “So you want to draw pictures? Okay, lets draw a picture. Here’s my box; I cut it into three. Here’s another box I cut it into three. Now what?”

“You color in three, then you color in two,” answers a student. Tony does this at the board. He continues, “So now I have five sixths. Is that correct?” One student is heard saying, “Yes.” Then another student answers above the murmur of the others, “No, you have five thirds.”

Tony tells the class, “Remember the bottom number tells us how big they are. Each of these pieces are thirds. So remember when you add fractions add the numerator and leave the denominator alone.” The class does one more problem, then they recite, “Add the numerator, and leave the denominator alone.”

The class ends with students working together on an additional challenge to the cookie problem. Because of time, Tony moves quickly through each item, asking the class if each item needs to be purchased. Students can find the answer to most of the items mentally. They pause and work out butter and white sugar easily, but when they get to the walnuts there is some confusion. Tony draws a picture so students can solve this. He concludes the class with a reminder of the rule, “Add the numerators, leave the denominator alone.”
Tony readily addresses student misconceptions, and his explanations are conceptually based. However, he does not provide the students the opportunity to express their own ideas. Student communication is not always conceptual in nature. Although some of the interaction is conceptual, this type of interaction is not consistent. One group is given a conceptualized task with high cognitive demand; but when the students have trouble accessing the task, Tony steps in and guides them to the solution. A detailed summary analysis for Observation Eleven can be seen in Table 16.

Observation Twelve - Grade Seven

There seems to be only one grouping of seventh-grade students. The students are quietly seated at tables. Tony introduces the topic to be covered, “Today we’re going to cover an important topic in geometry; equivalence and similarities.” His introduction is followed by a quick review of related topics. “What does the word congruence mean in geometry?” Tony asks. A student mumbles, “Exactly, the same.” “They both have the same measurement.” “The angles are the same.” Tony uses these answers to clarify the definition of congruence, “Yes the angles are the same. And if we’re talking about two shapes not only are the angles the same but also the lengths of the sides are the same.”

“So, for example, let’s say you have a square (He draws the shape on the white board) that’s four inches on the side. By definition a square has four right angles.” A student interrupts and Tony moves on with his example, “In order for another shape to be congruent to this (the original shape) what has to be true?” Mary says and Tony echoes as he draws a second shape on the board, “All angles are the same and all sides are the same.”
Table 16

*MQI Analysis for Observation Eleven - Grade Seven*

<table>
<thead>
<tr>
<th>MQI Dimensions</th>
<th>Segments</th>
<th>Scores</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>RICHNESS OF THE MATHEMATICS</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Linking Between Representations</td>
<td>S1 – S4</td>
<td>N</td>
<td>There was little use of multiple representations when Tony is explaining the work to be done. Tony used pizza to represent a fraction of 3/8 and explained the meaning of numerator and denominator.</td>
</tr>
<tr>
<td></td>
<td>S5</td>
<td>H</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S6</td>
<td>M</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S7</td>
<td>M</td>
<td></td>
</tr>
<tr>
<td>Explanations</td>
<td>S1 – S4</td>
<td>N</td>
<td>While working with the cylinder group Tony spent much more time explaining procedures and not concepts.</td>
</tr>
<tr>
<td></td>
<td>S5 – S7</td>
<td>H</td>
<td></td>
</tr>
<tr>
<td>Mathematical Sense-Making</td>
<td>S1</td>
<td>N</td>
<td>Tony worked with students to help them see what numbers mean in the context of the recipe problem.</td>
</tr>
<tr>
<td></td>
<td>S2</td>
<td>M</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S3</td>
<td>N</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S4</td>
<td>N</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S5</td>
<td>H</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S6</td>
<td>M</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S7</td>
<td>M</td>
<td></td>
</tr>
<tr>
<td>Multiple Procedures or Solutions Methods</td>
<td>S1 – S7</td>
<td>N</td>
<td>No multiple procedures were used in this lesson.</td>
</tr>
<tr>
<td>Patterns and Generalizations</td>
<td>S1 – S5</td>
<td>N</td>
<td>For the most part students were not asked to look for patterns and generalizations. At the end of the lesson, students were asked to identify a pattern in adding fractions. This was a passing statement and not much time was given for students to express their thinking.</td>
</tr>
<tr>
<td></td>
<td>S6</td>
<td>L</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S7</td>
<td>N</td>
<td></td>
</tr>
<tr>
<td>Mathematical Language</td>
<td>S1</td>
<td>L</td>
<td>There was some use of mathematical language by teacher and students, but the use of mathematical language was not dense throughout the lesson. Tony asked a student what the word difference means. The student said it meant different. Tony explained to the student that in math difference means subtraction.</td>
</tr>
<tr>
<td></td>
<td>S2</td>
<td>L</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S3</td>
<td>N</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S4</td>
<td>N</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S5 - S7</td>
<td>L</td>
<td></td>
</tr>
</tbody>
</table>
Table 16—Continued

<table>
<thead>
<tr>
<th>MQI Dimensions</th>
<th>Segments</th>
<th>Scores</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>WORKING WITH STUDENTS AND MATHEMATICS</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Remediation of Student Errors and Difficulties</td>
<td></td>
<td></td>
<td>Except for the first section, Tony spent lots of time focusing on student understanding of the material. He asked students questions about their plan for figuring out the problem. Tony asked students to get help from each other. He seemed intent on making sure all the students understood what they are to do and how to do it.</td>
</tr>
<tr>
<td>Teacher Uses</td>
<td></td>
<td></td>
<td>Most of the student responses were short one-word answers. Tony elicits student contributions through many Inquiry-Response-Evaluation type of teacher talk. These segments there was no evidence of student communication. He did not use their contributions to further mathematical ideas. When students explained what they did, they provided procedures, not explanations, on why they did what they did.</td>
</tr>
<tr>
<td>Student Mathematical Contributions</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Student Provide Explanations</td>
<td>S1</td>
<td>N</td>
<td>There were some moments where students provided explanations. Students explained how they solved the non-routine recipe problem. Teacher did ask students to explain math to their peers, but the explanations were not on video.</td>
</tr>
<tr>
<td>Student Mathematical Questioning and Reasoning</td>
<td>S1 – S7</td>
<td>N</td>
<td>Teacher used mathematical questioning. Students were asked to reason mathematically.</td>
</tr>
</tbody>
</table>
Table 16—Continued

<table>
<thead>
<tr>
<th>MQI Dimensions</th>
<th>Segments</th>
<th>Scores</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Students Communicate</td>
<td>S1</td>
<td>N</td>
<td>After teacher prompting and questioning students made substantive</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>statements that added to the understanding of the math.</td>
</tr>
<tr>
<td>about the Mathematics</td>
<td>S2</td>
<td>L</td>
<td>Students provided some explanations on how they solved the recipe</td>
</tr>
<tr>
<td>of the Segment</td>
<td>S3</td>
<td>L</td>
<td>problem by presenting the steps of how the problem was solved.</td>
</tr>
<tr>
<td></td>
<td>S4</td>
<td>N</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S5</td>
<td>M</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S6</td>
<td>M</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S7</td>
<td>L</td>
<td></td>
</tr>
</tbody>
</table>

| Task Cognitive Demand  | S1       | H      | Tasks required more than just procedural solving of math problems.    |
|                        | S2       | L      | Although the recipe problem provided an opportunity for high student   |
|                        | S3       | L      | engagement in mathematical thinking. Tony provided so much support     |
|                        | S4       | N      | that the problem deteriorated into solving for a rote algorithm.       |
|                        | S5       | L      |                                                                      |
|                        | S6       | L      |                                                                      |
|                        | S7       | L      |                                                                      |

| Student Work with      | S1       | H      | Student reasoning was low because teacher provided lots of scaffolding.|
| Contextualized Problems| S2       | L      | He told the students that finding the area of a cylinder that they     |
|                        | S3       | L      | should find the area of the rectangle and the area of the circle,     |
|                        | S4       | L      | thereby doing the thinking for them.                                  |
|                        | S5       | L      |                                                                      |

“What we’re going to talk about today—what’s new is the idea of similar. I think of congruence is like identical twins. They look the same, they act same, and they have the same size.” He labels the two squares on the board as congruent. Then he proceeds to define similar.

“The idea of similar has a very specific definition in math. Two objects are similar if they have the same shape but are not necessarily the same size. Let’s just say I had a square that had all right angles, but it’s eight inches on the sides. That would be a similar object. Now, one of the things we do is to check and make sure it is the same
shape. We can find the ratio of the sides. Well, we said this one is four inches and this one is eight inches (As he points to the smaller square and then the larger square.). Well, if I were to divide this side (bottom side of the smaller square) by this side (bottom side of the larger square) what would I get?” Ted answers, “Two.” Tony tells the student that he’s got the numbers confused. The ratio he is looking for is four eighths not eight fourths. He writes $\frac{8}{4}/\frac{8}{8} = 1/2$. “Similarly, if we take this side (pointing to the left side of the smaller square) and divide it by this side (pointing to the left side of the larger square), what would we get?” Several voices answer, “One half.”

“If I do that with all the sides I should get the same ratio, the same fraction. That’s the definition of similar. All the angles are the same and the sides make a proportion…or in the same ratio.”

Tony put up some examples on the white board using his iPad, “The question is, Are these two things similar?” Someone mumbles, “No.” “Well, what two things do we need to check?” There are several answers given, “Sides.”

Tony creates a list on the board. Is it similar? (1) Congruent Angles, (2) Proportional sides. Tony tells the students that he checks the angles first. He tells his students that this is because it’s easier to see if the angles are congruent. He checks the angles of the two given shapes. Because all the angles are not congruent, he decides that the shapes are not similar. The students sit quietly looking at the board.

Tony moves on to the next set of shapes. He checks the angles of the two triangles given. The angles are congruent; so, he moves on to check the sides. He tells the students that it is important to make sure that they find which sides match up, because they can only compare matching sides. With little student input, he writes and calculates the
proportion of the side lengths of the two given shapes. He explains that sides are proportional, “When we divide the corresponding side to the shapes, we get the same simplified answer.”

Tony projects another set of shapes, two quadrilaterals. Then he asks, “Are these shapes similar?” He answers the question by identifying the congruent angles. He spends time describing how he makes sure that he is comparing the corresponding sides to the shapes. He also explains that it doesn’t matter which length is on top and which is on the bottom of the fraction “As long as you’re consistent”, he explains with how you set it up, keeping the top number from the same figure on each fraction. Finally, he proceeds to solve the proportion asking the students to identify the numbers, as he fills in the numbers on the board and then solves the problem.

Next, Tony directs the students to solve problems found in iPad e-books. The students are asked to solve two problems on their own. Tony walks around the room checking student work and helping students that need help. His conversations with students are procedural in nature. After four minutes, he asks the students to direct their attention to the board.

Tony tells the students that seventh-grade math is moving toward algebra, “Which is where you find unknown values.” The next problem gives students similar shapes and asks students to find the measure of the missing angle. He asks, “What do we know about similar figures, first of all?” Jim quickly answers, “They have congruent angles and they have proportional sides.” Tony responds that the book does not give us any information about the sides. “How can we use the fact that similar shapes have congruent angles to find the missing angles?” Emanuel answers but it is unintelligible on the video. Tony
asks, “How did you get that?” He is unable to explain how he got the answer. Jim says, “They have to be congruent so you have to flip it around. So, the angles match.”

Tony explains, “Do you notice that we have two right angles, an obtuse angle, and an acute angle. Well, which of these is that (pointing to the angle with the missing value).” He quickly matches the angles and tells the students that the unknown is 78.

Then he goes on to the next problem. This problem asks for the measure of the missing side. Tony reminds the students of how the previous problems were solved. Tony finds the matching sides, writes the proportion, and asks the students what “b” equals. In the same manner, Tony solves three problems with minimal student input. Then he asks the students to do the next two problems on their own. After about two minutes, he checks student answers as they leave for their next class.

The explanations Tony delivers in this lesson are complete and conceptual. He helps students make connections to see the mathematical patterns and generalizations. Student communication is mostly procedural. Student engagement in Common Core-Aligned Practices in limited. A detailed summary analysis for Observation Twelve can be seen in Table 17.

**Observation Thirteen - Grade Seven**

This is a seventh-grade class. The students seem to be attentive and engaged in the class activities . . . Judith, the teacher, asks the students to compare the difference between an expression and an equation using a Venn diagram. The students don’t seem to have any problem explaining what they think as Judith writes their responses on the Venn diagram. “Expressions have no equal sign. Expressions and equations require at least two
Table 17

MQI Analysis for Observation Twelve - Grade Seven

<table>
<thead>
<tr>
<th>MQI Dimensions</th>
<th>Segments</th>
<th>Scores</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>RICHNESS OF THE MATHEMATICS</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Linking Between Representations</td>
<td>S1 – S4</td>
<td>N</td>
<td>Although Tony used some representations to describe the idea of congruent and similar shapes, he did not use multiple representations.</td>
</tr>
<tr>
<td>Explanations</td>
<td>S1 – S3</td>
<td>H</td>
<td>Tony used the analogy of twins to explain the concept of congruent and similar shapes. He compared congruent shapes as identical twins and similar shapes are like siblings they look the same but they are not identical.</td>
</tr>
<tr>
<td>S4 J</td>
<td></td>
<td>N</td>
<td></td>
</tr>
<tr>
<td>Mathematical Sense-Making</td>
<td>S1 – S4</td>
<td>N</td>
<td>Tony did not use multiple procedures or solutions methods in these segments.</td>
</tr>
<tr>
<td>Multiple Procedures or Solutions Methods</td>
<td>S1 – S3</td>
<td>H</td>
<td>Tony helped students see patterns. He showed students similar shapes and congruent shapes, and then spent time pointing out the features that make two shapes congruent and two shapes similar.</td>
</tr>
<tr>
<td>S4 J</td>
<td></td>
<td>N</td>
<td></td>
</tr>
<tr>
<td>Patterns and Generalizations</td>
<td>S1 – S3</td>
<td>H</td>
<td>Tony helped students see patterns. He showed students similar shapes and congruent shape, and then spent time pointing out the features that make two shapes congruent and two shapes similar.</td>
</tr>
<tr>
<td>S4 J</td>
<td></td>
<td>N</td>
<td></td>
</tr>
<tr>
<td>Mathematical Language</td>
<td>S1</td>
<td>H</td>
<td>Tony was careful to use the appropriate language to identify shapes, angles, and similar and congruent shapes. Tony drew attention to the idea that the word similar has a very special definition in math.</td>
</tr>
<tr>
<td>S2</td>
<td></td>
<td>H</td>
<td></td>
</tr>
<tr>
<td>S3</td>
<td></td>
<td>L</td>
<td></td>
</tr>
<tr>
<td>S4</td>
<td></td>
<td>N</td>
<td></td>
</tr>
<tr>
<td>WORKING WITH STUDENTS AND MATHEMATICS</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Remediation of Student Errors and</td>
<td>S1</td>
<td>N</td>
<td>Tony proceeded in a step-by-step fashion explaining the concepts to students, and he stopped to mediate any time he became aware that a student did not understand. As students worked individually he walked around and helped students that were not solving the problems correctly.</td>
</tr>
<tr>
<td>Difficulties</td>
<td>S2</td>
<td>N</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S3</td>
<td>M</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S4</td>
<td>N</td>
<td></td>
</tr>
<tr>
<td>MQI Dimensions</td>
<td>Segments</td>
<td>Scores</td>
<td>Notes</td>
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<tr>
<td>------------------------</td>
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</tr>
<tr>
<td>Teacher Uses Student</td>
<td>S1</td>
<td>L</td>
<td>Tony acknowledged student pro forma contribution such as figuring out if two shapes are congruent or similar by comparing sides and angles after he taught students how to setup proportion and solve for x.</td>
</tr>
<tr>
<td>Mathematical Contributions</td>
<td>S2</td>
<td>L</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S3</td>
<td>H</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S4</td>
<td>L</td>
<td></td>
</tr>
<tr>
<td>Students Provide</td>
<td>S1</td>
<td>N</td>
<td>Tony asked students to explain how they can use their mathematical knowledge they should find the missing angle of two given similar shapes. When a student gave the answer Tony asked asks, “How did you get that?” A student explained why it is possible to find the missing angle in of two similar shapes.</td>
</tr>
<tr>
<td>Explanations</td>
<td>S2</td>
<td>N</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S3</td>
<td>L</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S4</td>
<td>N</td>
<td></td>
</tr>
<tr>
<td>Student Questioning and Reasoning</td>
<td>S1</td>
<td>N</td>
<td>A student asked if the variable is always on top on the proportion. This question indicates that she was looking for a generalization to create an algorithm.</td>
</tr>
<tr>
<td></td>
<td>S2</td>
<td>N</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S3</td>
<td>L</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S4</td>
<td>L</td>
<td></td>
</tr>
<tr>
<td>Student Communicate about the Mathematics of the Segment</td>
<td>S1</td>
<td>N</td>
<td>A student provided substantive contributions explaining how to find the missing length of two similar shapes in the third segment.</td>
</tr>
<tr>
<td></td>
<td>S2</td>
<td>L</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S3</td>
<td>M</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S4</td>
<td>L</td>
<td></td>
</tr>
<tr>
<td>Task Cognitive Demand</td>
<td>S1</td>
<td>N</td>
<td>Tony lead student through solution steps therefore they are not required to do the thinking. One student told him he figured it out while Tony was trying to teach the class.</td>
</tr>
<tr>
<td></td>
<td>S2</td>
<td>N</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S3</td>
<td>L</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S4</td>
<td>N</td>
<td></td>
</tr>
<tr>
<td>Student Work with Contextualized Problems</td>
<td>S1</td>
<td>N</td>
<td>In most of the segments, Tony changed this routine problem by giving students a procedure for solving that they need to use to figure out the problems. It’s only when he asked how they would solve for x in the proportion problem that students need to use mathematical knowledge to do the work. But there were only two students that answered answer. The first only provides the answer and Tony moved on to the next student how explained can use the proportion and solve for x.</td>
</tr>
<tr>
<td></td>
<td>S2</td>
<td>N</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S3</td>
<td>L</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S4</td>
<td>N</td>
<td></td>
</tr>
</tbody>
</table>
numbers or variables.” “Expressions have an answer.” Judith adds on to the student statement by providing the word solve, and writes solve on the board. Then she asks students what you call it when you work with an expression. The response comes slowly, but several students answer, “Evaluate.” When the students seem to run out of ideas, Judith asks questions to prompt them. “With expression and equations do they both involve operations of some kind?” Several students answer, “Yes.” Judith adds this idea to the Venn diagram. “How many expressions are involved in an equation?” Again, several students answer, “Two.” Judith writes on the equation side of the diagram, “Two or more expressions.”

Judith very quickly, almost abruptly, changes the conversation. She tells the students that they will be playing a game. There is lots of drama in her voice, as she describes the game as beginning very competitive and proceeds to pick the teams. “For Team A, I would like these three boys. Would you come up here and be Team A.” Clenching her fists and moving them up and down she again emphasizes that this is a really, competitive game. Moving her attention to the other side of the room, while the students in Team A come to the front of the classroom, she asks three students to stand in the front of the classroom across from Team A. These students will be Team B. Again, she emphasizes how competitive this game will be. With a voice that sounds like she is rethinking her team choices, she asks two more students to join Team B. “Oh! Can you three right here join Team B?”

As Team B gets bigger and bigger, the students in Team A begin to get a look of concern; one student points to the other team with a quizzical look on his face. Ignoring
Team A’s concern, she reminds the student that this game is competitive and adds more students to Team B.

Finally, Charlie, one of the three students on Team A interjects, “That’s a huge team!” Judith restates the students concern and adds, “What’s wrong with that?” Charlie continues to challenge the size repeating, “That team is huge.” A comedian on the other team baits the other team, “Let’s play tug-of-war,” in a joking manner.

In response to the students’ concern, Judith asks one student from Team A to sit down and switches one student from Team B to Team A. Then she asks, “Is that better.” The class seems to be enjoying what’s going on. “Um, no,” Charlie says with a tone of mild frustration. Judith adds another student to Team B. Charlie, interjects, “That’s not helping.” Judith counters, “Why?” With a tone of growing frustration, Charlie answers, “Because you’re adding to them and not us.” Judith follows this by having two students from Team A sit down and all the boys move to team B. By now the students have figured out that there is something Judith is trying to teach. They are enjoying the drama between Charlie and their teacher. Team A now has only two girls and Team B has six students.

Judith turns her attention to a student who is sitting and asks Lily to pick a team. Wendy chooses Team A, the smaller team. Judith queries why she has chosen this team. In the spirit of the game Wendy responds that Team A is better because it only has girls. Everyone laughs. Judith asks, “Really, why do you think you should choose Team A.” Wendy answers, “They have less students.”

Judith sends the students back to their seats now that it is obvious that she has made her point about balancing sides on teams. As the students return to their seats
everyone is smiling. Judith debriefs with students, “According to that game the two sides were not ____?” Several students shout-out answers, “Even, Equal, Balanced.”

Judith tells the students that they will be discussing balance and equations. She reminds students that in the game Charlie had wanted the side to be balanced.

Judith directs the student’s attention to her desk where she has set up a *Hands-on Equation* © balance platform using a small tin bucket to represent the variable. She has checker chips to represent one. She rattles the small tin bucket to show students that there are some chips in the bucket. Then she places the bucket and three chips on the side of the balance platform and places seven chips on the other side of the scale. She tells the students, “Right now this is my equation and it is balanced,” and she labels the bucket as *n* for the variable.

Judith asks the students, “What is something I can do to keep my equation balanced?” After a long pause, Jack puts forth an idea. “You can take one chip from each side of the balance platform,” he says. Judith removes the one chip from each side, and then asks if there is anything else that can be done to the equation and keep it balanced. After a long pause, another says, “You can keep doing the same thing.” Judith has the student explain what that means. Lauren says you can take one more from each side.

Elizabeth says that you could also add chips to each side. Judith agrees with her, but gives the students this bit of new information, “My goal is to get nothing else with it. So, I can say my bucket equals however many are over here.” She points out that currently on the balance scale the contents of the bucket plus one equals five, because there are five chips left on the other side of the balance scale from the bucket.
“What do I need to do to find out about what’s in the bucket?” she asks. Peter answers, “You need to take one more (chip) off.” Judith agrees then asks the students, “Could you have done it in one step?” The class agrees and Judith asks them to state the problem. Wendy answers by saying that they could have taken three from each side.

Judith writes an equation \( N + 3 = 7 \), and tells the students that now they will help her create another problem. She writes \( N + 5 \) on a card and 10 on another card and places them on either side of the balance scale. “Hum, how do I build this using the chips and the bucket?” she asks. “What’s one thing I can do to build the equation?” Georgiana says, “You could add ten chips to that side.” pointing the side with the equal ten on it. Judith adds ten chips then asks another student what to do next. Elizabeth says that you could put the bucket on the balance scale. With a tone that sounds as if she has forgotten, Judith asks, “What does the bucket represent again?” Elizabeth replies, “The variable.” Judith expands, “We don’t know what’s inside the bucket.” Elizabeth continues, “Then you would add five to the same side.” Judith asks for clarification, “Inside the bucket or outside the bucket?”

Now that the equation has been setup, she asks the class, “What can we do to find out how many chips are in the bucket?” Paisley responds, “Take one from each side.” Judith follows her instructions. Darren, the next student called replies, “Take four more from each side.” Judith looks to the class for confirmation that Darren’s instructions are correct. Once the class assents, she takes four from each side. She reminds the students that they have taken five from each side then asks, “How many in the bucket?” The students chorus, “Five.” Then she asks the students if that works with the equation
written on the balance scale, “Five in the bucket plus five, does that give you ten?”

Again, the students agree.

Judith tells the students that they are going to work on three problems using the chips and a balance drawn on their tabletop. She hands out chips to groups of two students and writes the first equation on the board, \( M + 3 = 9 \), so students can get started. The students get right to work. There is a buzz in the room. Judith walks around the room asking students what they are doing. Soon you can hear different groups saying, “We got it!”

When each group is finished, Judith asks for a group to volunteer to explain to the class what they did. The group that comes up—three girls—begin their explanation, “Where this (pointing to the bucket) represents \( M \). Then you add three (chips). The equation says equals nine so you must add nine chips to the other side. So you are able to find out what’s in the bucket you should take out three.” (They take three from each side.) “So, in the bucket is six.”

Judith asks the class, “Does that make sense, six plus three equals nine, when you put six into the bucket. When you evaluate \( M \)?” The class responds, “Yes.” The girls sit down to applause and bravos.

After thanking the girls, Judith writes another equation on the board, \( 4 = x + 1 \). With a buzz of conversation and clinking chips the students get right to work. Judith commends a group for solving the problem quickly. After giving students a few more seconds to work on the problem, she asks the students to raise their hands if they have finished. It seems that the groups are finished, so Judith asks for their attention to the
front of the class. She counts backward three, two, one; to encourage student’s attention forward.

The students quickly settle, and Judith asks what the action was they performed to figure out what x equaled. She restates the question, “What did you add or subtract from each side so the equation remained balanced?” A student answers, “We took away one from each side.”

Judith surmises that the students understand the concept. As she walks to her table she tells the class, “Now I have something different for you to try. It seems you understand this concept. What would you do if the equation had you add something to a variable?” After a minute pause, several students answer, “You subtract,” in chorus.

Judith continues, “So if I have—Are you ready for this? You must pay close attention—a bucket and I take four out of the bucket. I have four in my hand. And my equation equals two. (x - 4 = 2). How can I find out what’s in my bucket? I have four of them in my hand.” She moves her hand toward the bucket as if to put four chips into the bucket. Several students respond, “Put them in.” Judith directs her question toward, Warren, saying “What should I do?” Warren is unsure, he smiles, then fidgets, then in a very soft voice answers, “Put them back in.”

“What else do I have to do? A bucket with four missing is equal to two. If I put four back in the bucket, what else do I must do? How many chips do I have to put on the other side?” asks Judith. Several students voice reply, “Two.” Judith replies to their answer by gesticulating toward each side of the equation with four chips in each hand. She continues supporting student understanding by adding, “I don’t know how many are in the bucket but I put four back. Then what do I have to do here? (Pointing to other side
of the equation.) I have to add four here also. What is in my bucket?” Several students answer, “Six”. Some sound hesitant, so she counts the chips. Then asks if students are ready to try one with her.

“I want you to think of how you are going to set the bucket up if I have \(x - 3 = 6\),” as she labels the balance scale. She asks the students to figure it out in their groups. Judith walks around the room listening to the students as they work. She complements students for their good work. After a bit, she asks the students to bring their attention to the front of the class. She comments, “I see that several of you have the six on the right side of the equation. If that’s the case, I have three in my hand. What do I add to the bucket?” As a student answers, “Three,” Judith continues, “If I add three to the bucket then I have to add three to the other side of the equation. So, what does \(x\) equal?” Judith and the students answer this question together with a sense of satisfaction, “Nine.”

As she wraps up the class, Judith tells the students that they will continue to work with equation solving strategies the following day. Next, she reviews the main ideas covered in this lesson. She asks all the students to make teams that are balanced. The students get up and begin discussing how they will arrange themselves. Some students are busy counting while others are directing their peers where to go. Soon the students have grouped themselves.

Judith says, “I heard some of you saying, ‘make the teams even;’ but ‘even’ already has a definition in math. So, we don’t say that an equation is ‘even,’ we say that an equation is ‘balanced.’ You all did a very good job today. For your homework tonight, I am going to write an equation on the board. You are going to write a note, an email to a
friend that doesn’t understand. You’re going to tell that friend how to solve that problem.”

As students disperse back to their seats, they create some commotion. Judith reminds them of the correct procedure by asking them what they are supposed to be doing right now, with an emphasis on “right now.” Arianna explains that they should be copying their homework. Judith asks for further clarification of their assignment. Another student says, “Write an email.” One student asks if it should be an email. Judith writes the equation on the board as she re-explains that students may use any form of written communication to explain how to solve the equation. The students pack up and class is dismissed.

This lesson the Richness of the Mathematics is strong and consistent. Students use multiple representations to show how to solve for \( n \). Students make sense of mathematical ideas, create generalizations, find patterns, and use mathematical language to describe their understanding. The students make few mistakes, and their communication is conceptual. The students participate in Common Core-Aligned Practices. They reason about solution methods discovered in the lesson. The task engaged the students to think mathematically. A detailed summary analysis for Observation Thirteen can be seen in Table 18.

**Observation Fourteen - Grade Eight**

This observation took place in the spring. This eighth-grade class has been separated by ability; it is composed of students who were not ready to take Algebra I. This is a Pre-Algebra class. The class is using a pre-algebra textbook called *Transitions to Algebra: A Habits of Mind Approach* (Mark, Goldenberg, Fies, Kang, & Cordner, 2015). This book
Table 18

*MQI Analysis for Observation Thirteen - Grade Seven*

<table>
<thead>
<tr>
<th>MQI Dimensions</th>
<th>Segments</th>
<th>Scores</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>RICHNESS OF THE MATHEMATICS</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Linking Between Representations</td>
<td>S1</td>
<td>N</td>
<td>Through teacher talk, analogies, and student explanations mathematical concepts were developed.</td>
</tr>
<tr>
<td></td>
<td>S2 – S4</td>
<td>H</td>
<td></td>
</tr>
<tr>
<td>Explanations</td>
<td>S1 - S4</td>
<td>H</td>
<td>Through teacher talk, analogies, and student explanations mathematical concepts were developed.</td>
</tr>
<tr>
<td>Mathematical Sense-Making</td>
<td>S1 – S4</td>
<td>H</td>
<td>Teacher called attention to how procedures solutions made mathematical sense.</td>
</tr>
<tr>
<td>Multiple Procedures or Solutions Methods</td>
<td>S1 – S4</td>
<td>N</td>
<td>Teacher did not point out multiple procedures or solution methods.</td>
</tr>
<tr>
<td>Patterns and Generalizations</td>
<td>S1 – S4</td>
<td>H</td>
<td>Teacher helped students develop generalizations about how to keep equations balanced.</td>
</tr>
<tr>
<td>Mathematical Language</td>
<td>S1 – S4</td>
<td>H</td>
<td>Teacher carefully provided mathematical terms and encouraged students to use them in their explanation.</td>
</tr>
</tbody>
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<thead>
<tr>
<th><strong>WORKING WITH STUDENTS AND MATHEMATICS</strong></th>
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</tr>
</thead>
<tbody>
<tr>
<td>Remediation of Student Errors and</td>
<td>S1</td>
<td>N</td>
<td>Teacher addressed in minimal student misconceptions and used questions to get students to better understand the concepts.</td>
</tr>
<tr>
<td>Difficulties</td>
<td>S2</td>
<td>M</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S3</td>
<td>N</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S4</td>
<td>N</td>
<td></td>
</tr>
<tr>
<td>Teacher Uses</td>
<td>S1</td>
<td>H</td>
<td>Students engaged in whole class discussion or in small group discussion to figure out the problem and explain their solutions.</td>
</tr>
<tr>
<td>Student Mathematical Contributions</td>
<td>S2</td>
<td>L</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S3</td>
<td>H</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S4</td>
<td>L</td>
<td></td>
</tr>
</tbody>
</table>
Table 18—Continued

<table>
<thead>
<tr>
<th>MQI Dimensions</th>
<th>Segments</th>
<th>Scores</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>COMMON CORE-ALIGNED STUDENT PRACTICES</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Student Provide Explanations</td>
<td>S1 – S3</td>
<td>H</td>
<td>Students explained how to solve equations with one variable using addition and subtraction.</td>
</tr>
<tr>
<td>Student Mathematical Questioning and Reasoning</td>
<td>S1</td>
<td>H</td>
<td>Students reasoned about how to balance an equation.</td>
</tr>
<tr>
<td>Student Communicate about the Mathematics of the Segment</td>
<td>S1 – S4</td>
<td>M</td>
<td>Throughout the lesson students explained their thinking either in small groups or class discussion.</td>
</tr>
<tr>
<td>Task Cognitive Demand</td>
<td>S1 – S4</td>
<td>H</td>
<td>Students made relationships between balancing teams and balancing an equation.</td>
</tr>
<tr>
<td>Student Work with Contextualized Problems</td>
<td>S1 – S4</td>
<td>H</td>
<td>Students applied their knowledge of math to solve equations using addition and subtractions.</td>
</tr>
</tbody>
</table>

is designed so students develop mathematical thinking habits while understanding algebraic concepts. The book uses puzzles first to allow students to problem-solve several cases, and then generalize how the puzzle was solved. Once students are successful with the representational puzzles, they are taught the mathematical short-hand to use to describe their ideas. The class has already started. They are participating in a whole class discussion. The students are seated in cooperative working groups of three or four students. They have sorted fractions into three categories $x < 1/4$, $x = 1/4$, and $1/4 < x < 1/2$. They already have sorted the numbers and Judith is asking them if there are any numbers that are not in the correct category. She looks at the board quizzically, specifically looking at the $2/10$ placed in the $1/4 < x < 1/2$ column. Clyde points out two
sixths in the third column. Judith continues, “Two over six, what does that equal?” The class choruses, “One third”

“That’s an equivalent fraction to one third.” She restates, “So does that belong there?” The class with a resounding voice says, “No.” Judith chooses to pause. Several students speak up at the same time. It’s not clear what anyone is saying. Judith asks Gilberto to come to the board and explain why he doesn’t think two sixths should be in this category. Before he comes up she stipulates to the class, “We’ll say that two sixths = one third.”

Gilberto: “Well to me no because all I see is one fourths and one half. (Pointing to the board.)

Judith checks her understanding of Gilberto’s statement, “So you think if it doesn’t equal one fourth or one half it can’t be in the category?” She continues, “What about if we thought of a number line?” She draws a number starting with zero, next one fourth, then another three fourths.

Then she explains this is a weird number line because the middle section is what equals one fourth. Judith asks, “Do you still think that one third would go in a different category?” Gilberto changes his mind. Now she wants to know why he has changed his mind. “I’m just guessing,” Gilberto replies. She now asks for someone to help Gilberto determine one way or the other.

During this whole discourse with Gilberto, the rest of the students are respectfully paying attention. Gilberto’s confusion is not met with any derision. Betty offers to help.

Betty: “Two sixths is less than one half because one half is three sixths and two sixths is less than that.”
Judith: “So you’re saying that because it’s less than three sixths it definitely belongs on this chart. Can someone determine if two sixths, which is equivalent to one third, should be in that category or a different category? Do we all agree that it’s not one fourth? Are we good with that? (Student says yes.) Can anyone explain where two sixths would be? Joshua.”

Joshua: “In the first category.”

Judith: “Why do you think it should be over here?” (Pointing to the first category.) You think it’s less than one fourth. What makes you determine it’s less than one fourth? (After a short silence.) Daniel, do you have something to add?”

Daniel: “I think one third is greater than one fourth.”

Judith: “Why do you think it’s greater than one fourth?”

Daniel: “Because the denominator is less numbers than the other.”

Judith: “We noticed a pattern earlier; the larger the denominator the smaller the number when there’s a one in the numerator. So, I think what you’re trying to say is that one fourth would fit somewhere in here. (Pointing to the middle section of the number line.) And one third would be somewhere in here. (Pointing to the last section of the number line. And one half would be somewhere in here. (Pointing to the area past one half on the number line.) Does that make sense according to the pattern we learned a while ago?” (Several students agree.)

Judith: “Another way we could do it is to use the decimal equivalent. And what is the decimal equivalent of one fourth. (Students answer 0.25) What is the decimal equivalent of one third? (Student answer 0.33) Judith corrects 0.3 repeated. “So, point is here,” (Pointing to the third section of the number line,) “and point two-five is in this next
section.” Pointing to the middle section.) “So, one third does belong there.” (Isaiah says, “Oh.”) “Are there any others that might need to be changed?” (Long silent pause.)

Judith: “So, what helped you determine what should be in each category? Who can tell me the pattern that you see here or how you can determine what else could go in this category?”

Two students work together to answer the question; although it is not audible on the tape, Judith thanks them. Judith now moves to the first section of the chart and asks students to give a generalization for numbers in the first section of the chart. Layna notices that the denominator is bigger than the numerator. Judith presses to get more details, “Is it a lot bigger or a little bigger?” Another student answers, “Bigger.”

Judith drops that line of questioning and asks for generalization about the last chart section, less than one half. Peter just repeats the section title, “It’s greater than one half”. “How can you tell,” asks Judith. Peter refines his answer, “All the numerators are a number less than one half of the denominator.” Looking around she asks, “So what keeps them out of these categories?” as she points to the beginning of the chart. “They’re greater than one fourth.” Judith adds to his incorrect generalization, by telling the student that the numbers in the first section have numerators that are less than one fourth of the denominator.

She ends this segment of the lesson by telling the class that they looked and found patterns and that today’s lesson would be about finding patterns when multiplying fractions. She continues to explain that they will be using the area model to multiply fractions. She asks for a volunteer to plot two thirds on a number line and she draws a blank number line on the board.
Gilberto volunteers. As he walks toward the board he asks, “Where is zero on the number line?” Judith tells him he must work it out for himself. He places zero at the left side and one on the right side of the number line. “I know that two thirds is in the middle of zero and two thirds,” he says. He continues, “I know that between zero and one is one-half. I know that one third is less than one half somewhere around here.” He places a line about between zero and one half somewhat closer to one half and labels it one third and then draws a line right next to one third and labels it two thirds.

Judith thanks Gilberto and asks, “Is two thirds bigger or smaller than one half?” The class choruses, “Bigger.” Judith asks Gilberto if he wants to change something. As he is returning to the board she adds, “It looks like you discovered that once you got back to your seat. He keeps one third where he originally placed it and moves two thirds to the other side of one half, and returns to his seat.” Judith then asks the other students if anyone wants to adjust the placement of one third and two thirds.

Sarah volunteers but doesn’t want to go up to the board. Judith tell the class to listen to what Sarah must say. Sarah asks, “Isn’t three-thirds one whole?” The other students agree. As Sarah steps to the board, Judith tells Isaiah that his work placed the fractions generally in the correct location. Sarah looks at the number line, then asks another student for help. Daniel comes up. He erases the one half on the line. Daniel then instructs Sarah to divide the number line into three sections. Daniel writes three thirds over the one. Then they label Isaiah’s lines as one thirds and two thirds. Judith asks the students if the number placement is correct. They all agree that it seems close but not exactly where they belong. Judith thanks Isaiah for starting the problem and Sarah for completing the work. Sarah heads for her seat smiling.
The students do not seem worried that they get an answer wrong. They are willing to provide support to each other as they work, and express their different ideas without making fun of anyone. This environment seems emotionally safe where students can work out their understanding of mathematical concepts.

The next step in solving this problem is discovering what one half of two thirds is. Judith tells the class to discuss this within their table groups for thirty seconds. As the students are discussing, she hands out dry erase markers for students to use on the tabletops. One group calls out they’re done. Judith heads toward them and has them explain how they solved the problem. The whole class is engaged figuring this out, as she checks in with each table. The discussion lasts about four minutes. Then she asks one of the tables to explain their procedure.

The first two students are inaudible, but Judith re-voices their statement. They used the pattern they learned in yesterday’s lesson. Next, she asks Joshua to explain. As he walks up to the board the other students in his group coach him about how he should explain the problem. Out of screen shot Joshua draws his method on the board. Judith wants to know what makes his method different. A student calls out that the procedures are the same. Judith asks the students to be respectful of each other by listening quietly. She asks Joshua to explain the difference between the two solutions, but she gives him the opportunity to pass the explanation to someone else in his group. He acknowledges that they just used the algorithm. Judith asks if anyone else did it a different way.

They conclude that they all did it the same way. Building suspense, she asks the students if she could share how she did it. Before she starts she asks them, “Are you ready!” Using the previously drawn number line, she says, “Half of this much is this,”
(As she traces the number line to two thirds), “Is this much,” (As she then divides the two thirds line in half, landing on the one third spot). There is a moment of total silence as the students absorb what she has done. Then a ripple of giggles washes over the class as they realize how simple the problem was. They understood the algorithm, but they weren’t thinking about what they were trying to accomplish.

Next, she asks them to plot three fourths and find one third in groups on their desks. The groups get right to work. As the groups work, Judith checks in with each group. After about four minutes she asks a group to share how they figured it out. Zane and Mitch come up; Zane draws, while Mitch explains. “First because it’s three fourths we have to divide the number line into four parts. So then we took three fourths and used division.” Zane takes the marker and writes, $\frac{3}{4} \div \frac{1}{3} =$, and uses cross multiplication to solve the problem. Then marks his answer, one fourth, on the number line. Judith thanks him for sharing and tells him that now they’ll see what everyone thinks. She wonders aloud, “You’re missing a few key components that would help that make more sense.”

She asks another group to come up. Brittany comes up and redraws the number line, then sits down. Daniel comes up redraws the number line and adds one thirds and two thirds, then says, “I put one fourth, because (Counting back from three fourths) one, two, I know it’s one fourth.” The class giggles with Daniel.

Judith helps to clarify Daniel’s work. She has him erase one third and two thirds, and tells the class that while we are talking about thirds, we’re not looking at where one third is on the number line, we’re talking about a section of three fourths. A one third section of three fourths. So how many jumps are there from zero to three-fourths, Zane? Can you show the jumps on the board? Daniel shows three jumps and Judith continues,
“So where is one jump out of three located.” Daniel realizes that providing an explanation is more difficult than he thought. Judith encourages the class, “I’m really excited that you get up here and you try your best. You’re helping each other learn.”

“Open your book and try number three in your groups,” Judith tells them. The students get to work. She cautions them that she should hear talking. They are not working alone but in groups. Judith walks about the room conferring with the groups as they work. The students all seem to be engaged and working on the problem. When she calls the students to stop she reminds them how well they have been working and thanks them for their industry.

After about five minutes Judith calls for the class’s attention. While working on the previous problem one group was chosen to act out the “Thinking Out Loud” section of the lesson from the textbook.

The students read a script, which describes a conversation between three students figuring out what is one third of six sevenths. First Daniel draws and explains that he used a number line, and finds that one third of six sevenths is two sevenths. Next Mitch reads his part, “I think one third of six sevenths is six over 21. I can show you using an area model.” He draws as he reads his line, “Here is a one by one square, I’ll make thirds vertically and sevenths across. You can see there are 21 pieces shaded so the area of that rectangle is six over 21. Oh, I just realized that six over 21 is equivalent to two sevenths, so we got the same answer. I wonder if there’s a way to show that the answers are the same.” Philip is hard to hear, but he ends his statement with “I can show that six over 21 is equal to two sevenths.”
While Philip draws this out, Judith asks the class to make a generalization about what they have seen. Ruth says, “It helps show fractions.” Judith agrees. When the students completed their drawing, she moves to the board and re-voices what the students have done. Judith ends her explanation with, “It’s something you have already been doing with fractions, but this is something that helps you more deeply understand fractions.” The class ends as she gives them a homework assignment.

The teacher in this lesson used many of the Richness of Math strategies. She used Mathematical Sense Making, Patterns and Generalizations, and Mathematical Language. The teacher used Student Mathematical Contributions to move instruction forward. The use of Common Core-Aligned Student Practices was evident in the student discussion as they tried to place the fraction in the number line and as they debated methods and rationales for placement. A detailed summary analysis for Observation Fourteen can be seen in Table 19.

Summary

Generally, all the teachers in this study had orderly math classrooms where math instruction was happening. Most of the students seem to be attending to, and participating in the instruction. The teachers seemed to create an environment where students were confident to learn math.

The teachers used a variety of methods from concrete manipulatives, analogies, and explanations for students to understand the concepts begin taught. They paid close attention to students’ need for support and provided it without hesitation.

Higher-order thinking was not consistently apparent in most lessons and segments. In many instances, the teacher was doing the work of thinking and the students
Table 19

MQI Analysis for Observation Fourteen - Grade Eight

<table>
<thead>
<tr>
<th>MQI Dimensions</th>
<th>Segments</th>
<th>Scores</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>RICHNESS OF THE MATHEMATICS</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Linking Between Representations</td>
<td>S1</td>
<td>M</td>
<td>Students and teacher linked representations between number line and fractional representations.</td>
</tr>
<tr>
<td></td>
<td>S2 - S5</td>
<td>N</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S6</td>
<td>H</td>
<td></td>
</tr>
<tr>
<td>Explanations</td>
<td>S1 – S6</td>
<td>H</td>
<td>Teacher helped students explain where fraction should be placed on number line.</td>
</tr>
<tr>
<td>Mathematical Sense-Making</td>
<td>S1 – S6</td>
<td>H</td>
<td>Students contributed ideas to make sense of problem. Teacher called attention to how procedures solutions made mathematical sense.</td>
</tr>
<tr>
<td>Multiple Procedures or Solutions Methods</td>
<td>S1</td>
<td>M</td>
<td>Judith pointed out that the number line problem could be solved using fractions or decimals. Students are asked to explain how two solution methods compare.</td>
</tr>
<tr>
<td></td>
<td>S2</td>
<td>M</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S3 - S5</td>
<td>H</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S6</td>
<td>N</td>
<td></td>
</tr>
<tr>
<td>Patterns and Generalizations</td>
<td>S1 - S3</td>
<td>H</td>
<td>Teacher noted the pattern: the larger the denominator the smaller the number when there’s a 1 in the numerator.</td>
</tr>
<tr>
<td></td>
<td>S4</td>
<td>H</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S5</td>
<td>N</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S6</td>
<td>N</td>
<td></td>
</tr>
<tr>
<td>Mathematical Language</td>
<td>S1 – S6</td>
<td>H</td>
<td>Teacher consistently used vocabulary and encouraged students to use vocabulary during explanations.</td>
</tr>
<tr>
<td><strong>WORKING WITH STUDENTS AND MATHEMATICS</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Remediation of Student Errors and Difficulties</td>
<td>S1 – S4</td>
<td>N</td>
<td>There were few student mistakes; the teacher addressed student mistakes each one. Judith addressed student ideas. She asked them to explain their answers. She also asked other students to explain student thinking.</td>
</tr>
<tr>
<td></td>
<td>S5</td>
<td>H</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S6</td>
<td>N</td>
<td></td>
</tr>
<tr>
<td>Teacher Uses Student Mathematical Contributions</td>
<td>S1 – S6</td>
<td>H</td>
<td>Teacher strongly used students’ contribution during whole class discussion. She attended to group discussion and used their ideas to move understanding forward.</td>
</tr>
</tbody>
</table>
Table 19—Continued

<table>
<thead>
<tr>
<th>MQI Dimensions</th>
<th>Segments</th>
<th>Scores</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>COMMON CORE-ALIGNED STUDENT PRACTICES</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Student Provide Explanations</td>
<td>S1 – S6</td>
<td>H</td>
<td>Class worked in groups and cooperatively to solve and understand mathematical ideas.</td>
</tr>
<tr>
<td>Student Mathematical Questioning and Reasoning</td>
<td>S1 - S5</td>
<td>H</td>
<td>Throughout the lesson students explained why they placed fractions on a certain point of a number line.</td>
</tr>
<tr>
<td></td>
<td>S6</td>
<td>N</td>
<td></td>
</tr>
<tr>
<td>Student Communicate about the Mathematics of the Segment</td>
<td>S1 - S5</td>
<td>H</td>
<td>Students spent time discussing mathematical ideas both in class discussions and small groups. Student communication drove instruction.</td>
</tr>
<tr>
<td></td>
<td>S6</td>
<td>N</td>
<td></td>
</tr>
<tr>
<td>Task Cognitive Demand</td>
<td>S1 – S6</td>
<td>H</td>
<td>Students made sense of student placement of where 1/3 of 3/4 would be placed on a number line.</td>
</tr>
<tr>
<td>Student Work with Contextualized Problems</td>
<td>S1 – S6</td>
<td>H</td>
<td>Students spent most of the class time supporting and reasoning on about fraction placement on the number line.</td>
</tr>
</tbody>
</table>

were passive receivers. Although at the time of instruction they could answer conceptual questions related to the topic, one wonders if they will remember the concepts when presented with them in the future. One might say that the students seemed to be familiar with the concepts but did not have personal understanding.
CHAPTER 5

CROSS-CASE ANALYSIS

Introduction

In this cross-case analysis, I describe how this group of teachers’ mathematics instructional strategies align with the strategies found in the MQI rubric. Then I compare the similarities and differences found within and between grade levels of first – second, third – fifth, and sixth – eighth grades.

However, before we consider whether teachers and students are truly engaged with mathematical reasoning, it was important to assess if the class time was used for actual math activities. Several studies show that there is a positive relationship between time on task and student achievement (Baker, Fabrega, Galindo, & Mishook, 2004; Godwin et al., 2016). Of the 80 segments only one segment was not fully devoted to working on math. In that segment, the students were putting their math books, base-10 blocks, and white boards away.

MQI Analysis by Total Usage and Quality of Usage

The MQI is an instrument designed to enable observers to identify the quality of mathematics instruction (Kane & Staiger, 2012, p. 19). According to Cohen and Ball (2001, p. 75), instruction is executed at the intersection of the teacher, the student, and the content. The knowledge and skill the teacher brings, the prior knowledge and motivation of the students, and the type and quality of resources work synergistically to create
mathematics instruction. In this section, I describe the teachers’ instruction by the four dimensions and the 16 strategies in the MQI. Each section addresses the total usage as well as the quality of the usage.

Comparisons of the Four MQI Dimensions

The MQI instrument divides math instruction into four dimensions: Richness of the Mathematics, Working with Students and Mathematics, Teacher Errors and Imprecisions, and Common Core-Aligned Student Practices. When looking at Figure 5 it is important to understand that each dimension can be used concurrently and/or individually depending on the students’ needs, teacher knowledge and comfort level with each dimension, the standards taught, and the curriculum used. All comparisons are given as percentages, since the total usage for each dimension does not add up to the total number of segments.

This graph shows that the teachers used the three dimensions: Working with Students and Mathematics (71%), Richness of the Mathematics (70%), and Common Core-Aligned Student Practices (66%) of the time. It also shows that the teachers made very few mistakes throughout the eighty segments. Errors and Imprecisions were found in only 3% of the segments. Because of the low incidence of errors, I will not be discussing this dimension in the following sections.

In addition to reviewing the total usage of each dimension, the authors of the MQI also provided information to describe the quality of the mathematics instruction. The quality of instruction refers to the distinctive character of something or to its rank, level,
or grade. “Quality includes the presence, absence, degree and composition of these features . . . [Quality can also] refer either to an overall character or to a collection of
distinct characteristics” (LMT Project, 2011, p. 30). Each code on the MQI is given a quality level. Teaching strategies are evaluated as Not Present, Low, Mid, or High quality of usage (NCTE, 2012). “These scales were directly shaped by the influences [of the authors] reading of the research and policy literature; [their] experiences teaching mathematics to children and studying teaching, teacher education, and teacher knowledge; and [their] viewing of [original study video tapes] and other videotapes of classroom practice” (LMT Project, 2011, p. 34).

Figure 6 shows the quality of usage in percentages. These were calculated based on the 80 segments including the one segment that did not contain mathematics instruction.
When comparing the quality of usage, one sees that although the usage of each dimension is about the same, the quality of usage is not. The teachers were more skillful using the strategies in the Richness of the Mathematics dimension (50% high quality, Figure 6) than in the other dimensions. The Working with Students and Mathematics dimension has the least amount of high quality instruction (39%) and the greatest amount of low quality instruction (23%). When teachers used, the Common Core-Aligned Student Practices dimension, the quality of usage was high (41%) of the time. Although the quality of usage seems about the same for this dimension as for the others, it is important to remember that it was the least used dimension.

Comparison of Individual MQI Strategies

The four dimensions are further divided into the 16 specific MQI strategies (NCTE, 2012) used in my analysis. There are 6 strategies under Richness of the Mathematics dimension, 2 strategies under Working with Students and Mathematics, 5 strategies under Common Core, and 3 strategies under Errors and Imprecisions. As
mentioned earlier, the three strategies under Errors and Imprecisions will not be described in this section because of the low incidence (3%) of usage. The six strategies under Richness of Math are: Linking Between Representations, Explanations, Mathematical Sense-Making, Multiple Procedures or Solution Methods, Patterns and Generalizations, and Mathematical Language. These strategies describe how the teacher uses mathematical knowledge and pedagogy to provide students with instruction. The two strategies under Working with Students and Mathematics are: Remediation of Student Errors and Difficulties and Teacher Uses Student Mathematical Contributions. These strategies identify how the teacher supports student understanding and contributions to mathematics. Finally, the five strategies under Common Core-Aligned Student Practices are: Students Provide Explanations, Student Mathematical Questioning and Reasoning, Students Communicate about the Mathematics, Task Cognitive Demand, and Students Work with Contextualized Problems. These strategies identify student response and engagement to instruction. Figure 7, Figure 8, and Figure 9 show the total percentage of usage for each of the 13 teaching strategies found in the 80 segments the teachers in this study. The teachers used, Explanations in (80%) of the segments, Mathematical Sense-Making in (83%) of the segments, and Mathematical Language in (90%) of the segments (Figure 7). The teachers used Student Mathematical Contributions in (90%) of the segments (Figure 8), and Students Communicate about the Mathematics in (85%) of the segments (Figure 9). The strategies used in less than half the segments were Multiple Procedures or Solutions (43%, Figure 7) and Student Mathematical Questioning and Reasoning (45%, Figure 9).
Figure 7. Richness of the Mathematics Strategies Usage.

Figure 8. Working with Students and Mathematics Strategies Usage.
The MQI rubric allows us to see the total usage and the quality of that usage. By looking at the high quality, mid quality and low-quality usage we can compare how well the teachers enacted instruction. The quality of instruction refers to the distinctive character of something or to its rank, level, or grade (Boston et al., 2015).

Figure 10, Figure 11, and Figure 12 show a comparison of high, mid, and low quality of usage for each of the 13 MQI strategies. In general, all 13 strategies were used at a high level of quality rather than a mid or low level of quality. The most striking feature of this data is that the high-quality usage was more prevalent in the Richness of Math Dimension (Figure 10). Four of the six strategies in this dimension were used at a high level more than 50% of the time: Linking between Representations (51%), Explanations (69%), Mathematical Sense-Making (60%) and Mathematical Language (53%).

*Figure 9. Common Core-Aligned Student Practices Usage.*
**Figure 10. Richness of the Mathematics Strategies Quality of Usage**

**Figure 11. Working with Students and Mathematics Strategies Quality.**
In the Working with Students and Mathematics dimension, Teacher Uses Student Mathematical Contribution was also used at a high level of quality (53%, Figure 11). In the Common Core-Aligned Student Practices dimension (Figure 12), none of the five strategies was used at a high quality more than 45% of the time.

Figure 11 and Figure 12 show that Teacher Uses Student Mathematical Contributions (Low 31%) and Students Communicate about the Mathematics (Low 31%) were the strategies with the lowest quality usage. To get a high score in both these strategies, students must make substantive comments. The student comments, recorded in this study, were often procedural.

In the following section, I describe each of the 13 MQI strategies, again leaving out Errors and Imprecisions because there were so few mistakes made by the teachers (3%). Richness of the Mathematics focuses on the teachers’ ability to teach math,
particularly their use of various instructional strategies. This dimension describes the teachers’ relationship with the material.

**Richness of the Mathematics - Linking Between Representations**

The Linking Between Representations strategy captures where mathematical representations have at least two elements in common and the teacher or students point out commonalities or differences between the representations (Figure 7 & Figure 10). Linking Between Representations had total usage of 61% and quality of usage was high 51% of the time. When comparing the quality of representational links, the teachers were more likely to have a high quality of usage when linking different representations.

Mathematical ideas can be represented with concrete objects, pictorially by using graphs and models, or abstractly by using numerical notation systems, and mathematical shorthand of expressions and equations. Zoltan Dienes postulates in a 2007 interview (Siraman & Lesh, 2007), that understanding mathematic structures or ideas require that students use multiple concrete examples. Linking between different representations allows the student opportunity to understand math ideas deeply. “Multiple representations…serve not merely as illustrations or a pedagogical trick but form a significant part of mathematical content and serve as a source of mathematical reasoning” (National Research Council, 2001, p. 95). The teachers in this study used multiple representations and identified the connections between them in more half of the segments.

Following are some examples of how teachers in this study used links between representations. Jonatha used multiple representations when she used the story of the King’s feet and the carpenter’s feet as an analogy for the concept that measurement units
must all be the same size. Iris used a representation she called “quick draw.” In “quick draw,” numbers are represented with sticks and circles. The tens are drawn as sticks, and the ones are drawn as circles. This activity allowed students to link the Arabic place value system of writing numbers with the actual value of the number. Tony and Deborah used divided circles and squares to represent the value of fractions. Deborah had students use color-coded fraction strips to help students visualize equivalent fractions. Judith used a balance, poker chips, and a bucket to represent an equation with variables. She portrayed each side of the balance as one side of the equation and the bucket representing the unknown. At one point, she rattled the chips in the small bucket as to emphasize that there are some chips in the bucket, but we don’t know how many. According to research (Coll et al., 2012; Richland, 2010), the process of using different representations helps the student link what they know to new concepts. In addition, “the way in which a concept is first learned create[s] obstacles to learning in a more abstract way… overcoming such obstacles seems to be a necessary part of the learning process” (National Research Council, 2001, p. 99). The study teachers effectively used various representations and effectively linked them providing students an effective launch pad for understanding more difficult concepts.

Richness of the Mathematics - Explanations

The teachers in the study used explanations quite often (80%, Figure 7). They explained why a procedure works, why a solution method is appropriate, or why an answer is true. According to Nicole Seling (2016), the mathematical work of teaching includes making explanations. As an explainer of mathematics, the teacher provides justifications and reasoning for mathematical objects (e.g. numerical notations, values,
procedures, and symbols). Educators have been using explanations for many years. Ahmose penned the Rhind Papyrus, the first mathematics textbook, around 1650 B.C. It gave mathematical problems and their solutions (Siraman & Lesh, 2007). Just like Ahmose, the study teachers in this study are experts at providing students with explanations. They were also more likely to use high-quality explanations (69%, Figure 10) than low-quality explanations (6%, Figure 10).

Evidence of high-quality explanations can be seen in several segments of individual teacher instruction. Rosita explained how to use centimeter cubes to measure the volume of a container. Judith and Maria used questions to guide students to their explanations. Judith said, “So, what helped you determine what should be in each category? Who can tell me the pattern that you see here or how you can determine what else could go in this category?” Although she did not give an explanation, her questions led Jared to make a generalization, “All the numerators are a number less than one half of the denominator.” According to “Adding It Up: Helping Children Learn Mathematics” a review of the research edited by Kilpatrick, Swafford and Findel found that “mathematics instruction in the United States consistently emphasize[d] teacher explanation and student practice of teacher provided procedures” (National Research Council, 2001, p. 71). As with other American teachers, the teachers in this study excelled at giving students explanations.

Richness of the Mathematics - Mathematical Sense-Making

Sense-Making was also a strategy that the teachers in this study used quite often (83%, Figure 7). Mathematical Sense-Making gives meaning to numbers and shows the relationships between numbers and their context. Sense-Making provides connections
between ideas and representations. In some instances, Sense-Making is the same as explanations, but in others, it is not. While explanations may tell students why something is true, mathematical sense-make gives meaning. Alan Schoenfeld (1992, p. 334) describes Sense-Making as the ability to seek solutions, explore patterns, and create conjectures. The teachers used Sense-Making strategies with a high quality of usage 60% of the time (Figure 10).

Jonatha asked her students for the meaning of digits. “This is a one, but it really means a 10.” Throughout the entire lesson, she asked students to find the meaning of the digits. Iris also emphasized the meaning of place value as students subtracted a three-digit number with a zero as the center digit, which required regrouping. Deborah asked students to identify the characteristics of a fraction that equals one. Several times throughout the class period, Deborah reminded the students of different ways they can write fractions equivalent to one. “Sense-Making allows student to make useful connections between values, concepts, and algorithms” (National Research Council, 2001, p. 206). Study teachers provided students with consistent and high-quality opportunities to make sense of mathematics.

**Richness of the Mathematics - Multiple Procedures or Solutions Methods**

This strategy describes the teacher’s use of more than one procedure or solutions for a mathematical problem. When compared with other strategies in this dimension it was used the least (43%, Figure 7). Of the 43% total usage, 25% of the time the quality was high, while mid quality was used 16% of the time (Figure 10). Although this strategy was not used often, when it was used the teachers were more likely to use it effectively.
Multiple solutions allow students to discuss and compare their ideas and can be teacher led or student provided. The use of multiple procedures and solution methods was the least used instructional practice, which can bring mathematical depth to instruction (Baingolbali, 2011; Leva-Vaynberg & Leikin, 2006).

In Iris’ first-grade class, students explained how they added 50 to 20. One student explained that she used a “quick draw” of 5 sticks and 2 sticks, each stick representing 10. The next student described how she had used a number fact she already knew to help her figure out the answer. Sidney used five plus two equals seven, then added a zero. A third student used to count on by tens; she started with 20 and then counted to 50. Iris responded to each student and accepted his or her answers as valid ways of finding the solution to the word problem.

Deborah’s students had to figure out how much is one eighths plus five eighths. Working in their Go Math textbook, they all solved it by first coloring one eighths of a rectangle divided into eighths and then coloring five eighths of the same rectangle. Then as a group, they solved the problem using fraction strips. Deborah pointed out that both solution methods got the same answer.

In Judith’s eighth-grade class, there were three columns: one was labeled $x < 1/4$, the next was labeled $x = 1/4$, and the last one was labeled $x > 1/4$. They were to place a set of fractions in the correct column. Judith asked students to defend the placement of two sixths in the last column. She recommended the students might use a number line to defend their ideas. After some back and forth she then asked the students if they might use decimals to defend the placement of two sixths. During their discussion, students used two ways of determining the comparative value of fractions. The associative,
distributive, and commutative properties of mathematical operations allow for students to solve problems in various ways. The teacher must “have sufficiently flexible knowledge of arithmetic to evaluate the various student solutions, or describe additional solutions methods, to validate the correct ones and to correct (student) errors productively” (National Research Council, 2001, p. 97). Teachers in this study used this strategy less than half the time with quality of usage only one-fourth of the time. Students are deprived from getting a deep understanding and flexible use of math concepts when teachers do not provide or invite students to provide multiple procedures for problem solutions.

**Richness of the Mathematics - Patterns and Generalizations**

In the patterns and generalization strategy, the students and/or the teacher uses inductive reasoning to see commonalities in solution methods, the meaning of numbers, or the development of mathematical definitions from several examples. The strategy Patterns and Generalizations was used in 61% of the segments (Figure 7). Of this, 40% of the usage was of high quality, while 11% and 10% were mid and low quality respectively (Figure 10). “Pattern generalizing is a major factor for developing the algebraic thinking of children with particular importance for the development of the concepts of variable and function” (Tanisli & Ozdas, 2009, p. 1487).

Rosita had students add three numbers in a different order. Then she asked them what they noticed about the answer. The students developed a generalization that the answer does not change. Rosita then asked them to give her a rationale why this was so. One student explained that the answer stayed the same because they “hadn’t added or taken away anything.”
In Jonatha’s class, they worked with tens and ones. Jonatha had the students make several numbers using base-10 blocks. Then they examined how these numbers were notated. Jonatha pointed out how the similarities in tens and ones were notated. In this case, the teacher pointed out the generalization for the place value method of mathematical notation.

The ability to see patterns and generalize in mathematics allows students to make connections between concepts and avoid errors. In addition, an awareness of patterns and generalizations allows students to join mathematical ideas and concepts making what Liping Ma (1999) calls “knowledge packets”, thus making learning easier. The teachers used this strategy less than one-half of the time and, with limited quality.

**Richness of the Mathematics - Mathematical Language**

This code describes how fluently the teacher and the students use mathematical language. This strategy had the highest usage (90%, Figure 7). The quality of this code was high in 53% of the segments, mid in 21% of the segments, and low in 16% of the segments (Figure 10).

The mathematical language code includes teacher emphasis of technical language and student use of precise language. The teachers in this study consistently used mathematical language during segments in most lessons. They also encouraged students to use mathematical language during discussions. Understanding the specialized vocabulary of math allows students to better understand concepts (Dunston & Tyminski, 2013).

Here are several examples of teacher effectiveness using this strategy. Tony told his class, “Something to keep in mind; the top number is how many and the bottom
number is how big.” After he used the definition, he used the words denominator and numerator throughout the lesson. In the second observation, he also spent considerable time noting the difference between congruent shapes and similar shapes.

As Rosita explained how to divide fractions, she used the words, divisor and dividend. She encouraged students to use divisor and dividend when they described the procedures they used to solve the division problems.

Jonatha and Iris required students to use tens place and ones place when explaining the value of a number. When a student said, “Take away,” Iris corrected him and had him restate the problem using the word minus. Deborah and Maria asked students to identify denominators and numerators in fractions. One of Rosita’s lessons was primarily focused on the correct use of longer, longest, and shorter, shortest as she compared snakes during her lesson. Teaching mathematical vocabulary is, according to Hughes, Powell, and Stevens (2016, p. 16), “one way to support children and promote progressive understanding of mathematics is to use precise and accurate language embedded within teaching strategies that progress and generalize across standards and grades” (2016, p. 16).

Next, I will discuss the dimension “Working with Students and Mathematics.” This dimension focuses on the teachers’ ability to connect with students by addressing their errors and misconceptions. It will also address the teachers’ ability to use student communication to move instruction forward. This dimension addresses the teacher-student side of the instructional triangle.
Working with Students and Mathematics - Remediation of Students Errors

Working with Students and Mathematics dimension captures whether the teacher understands and responds to students mathematically. This dimension looks at whether the teacher responds to mistakes and contributions, but also how a teacher responds to student mistakes and contributions.

Teachers can address student errors in several ways - procedurally, conceptually or through pre-remediation. A teacher can remind a student of the next step when using an algorithm would be coded as procedural support. On the other hand, if the teacher addresses the source of the error, then it is coded as a conceptual remediation, and the quality goes up. Pre-remediation is provided when teachers anticipate student errors or misconceptions and plan accordingly. The teachers in this study provide both procedural and conceptual remediation, leaning slightly towards providing procedural rather than conceptual remediation. Remediation of Student Errors and Difficulties and Difficulties happened in about half of the total segments (51%, Figure 8). The quality of this remediation was high 25% of the time, mid 13% of the time, and low 14% of the time (Figure 11). According to Kingsdorf and Krawec error analysis provides teachers with direct access to student thinking and allows teachers to make informed decisions for direct instruction (Brown & Skow, 2016; Kingsdorf & Krawler, 2014).

Tony asked a student to find the difference. Tony could tell the student was confused, so he asked him, “What does it mean to find the difference?” The student responded that the numbers were different from each other. Tony provided the student with a mathematical definition of difference.
By using questions, Judith and Maria helped students use previous knowledge to understand new learning that was causing them difficulty. As the students were solving the problem, one third of 15, Maria stopped by a student’s desk. By asking step-by-step questions, Maria helped the student negotiate the solution.

When Warner came up to the board in Judith’s class, he was trying to place two thirds correctly on a number line. He drew a number line on the board as depicted in Figure 13. He drew a number line placing the zero at one end, the one at the other end, one-half in the center, one-third between the zero and one-half but much closer to the one-half and two-thirds between one-third and one half.

![Figure 13. Student’s Number Line Example with Incorrect Placement of Two-thirds.](image)

After he was done, Judith asked the class, “Is two thirds bigger or smaller than one half?” After the class had chorused the answer, Warner returned to the board and moved the two thirds to the other side of one half. At this point, Judith had provided procedural support. Next, she asked for volunteers to see if they could improve on Warner’s placement of two thirds. Lyndsey came up and said that three thirds equal one. Then she too got stuck. Then Edwin came up and erased the one half and then divided the number line into thirds. Then he labeled Warner’s one third and two thirds on the number line. With this added information, we have a rationale for where Warner placed the two thirds.
By allowing the students to work the problem out themselves, Judith gave them an opportunity to develop fractional number sense. High usage of this strategy was found in one-fourth of the study segments.

**Working with Students and Mathematics - Teacher Uses Student Mathematical Contribution**

This strategy provides information on how the teacher uses student contributions to move instruction forward. It doesn’t matter if the student contribution is verbal/written or procedural/conceptual. Student contributions can come in the form of questions, explanations, generalizations, or representations. Although, all the teachers used this code, some teachers used it more than others. In some classes, student contributions were the sole driver of the class discussion while in other classes student contributions only happened in response to close-ended questions. The students in this study provided some form of student discourse in 90% of the segments (Figure 8). The quality of usage was high 53% of the time, mid 6% of the time, and low 31% of the time (Figure 11).

In one study (Sims, 2008), the average American teacher produced 89% of all classroom discourse and 69% of mathematics discourse. The same study comparing teachers in a Chinese classroom produced 65% of all discourse and 31% of math related discourse. The MQI rubric did not account for student contributions by counting the production of discourse; it looks for the type and quality of student discourse.

Iris used student answers to give direction to the next question she would ask. “When given a subtraction word problem,” one of Iris’s students said, “You know it’s subtracting because the problem says the bigger number first.” Iris used his contribution and asked, “Does that rule always work?” The next student pointed out that rule does not
work on the garage problem. Iris accepted all those ideas and kept moving the students by asking, “What does the problem say?” At this point, Iris decided to move on to what she planned to teach without a complete resolution to the students’ comment, but this exchange shows how Iris used student thinking to move instruction.

After practicing adding three numbers on the board while she manipulated the counters, Rose asked the students to go back to their seats and use counters to solve three number addition problems. When she saw that they were not able to follow the procedure working on their own, she quickly regrouped them at the meeting area and solved the problem with together. She was responsive to their questions and changed her plans.

The following five strategies focus on the Common Core-Aligned Student Practices dimension. The common core Standards for Mathematical Practice focus on what it means for students’ to be mathematically proficient. These standards describe “student behaviors, ensure an understanding of the mathematics, and focus on developing reasoning and building mathematical communication” (Rutherford, 2015, p. 16).

**Common Core-Aligned Student Practices - Students Provide Explanations**

In this strategy, students provide more then procedural explanations; they describe why procedures work, what numbers mean, or otherwise attend to the meanings of numbers or procedures. Students provided explanations in 61% of the segments (Figure 9). Forty-five percent of student explanations were of high quality, while, 5% usage was mid quality and 11% usage was low quality (Figure 12).

When students provide explanations, they demonstrate conceptual understanding of mathematics. “Student explanations can provide learners the opportunity to work through their understanding and learn from ideas of others” (Pan, 2014, p. 64).
In this study, teachers used a variety of ways to elicit student explanations. Maria asked students questions about their choices in solving the problems. Iris listened attentively; while students described problems then asked them, “Why do you think that is so?” Often, teachers re-voiced student explanations that were unclear. In comparison to teacher explanations which are found in 80% of the segments, and student explanations were only found in 61% of the segments.

**Common Core-Aligned Student Practices - Student Mathematical Questioning and Reasoning**

This strategy examines student reasoning beyond an explanation and makes statements about patterns or general rules related to the topic. Mathematical Reasoning and Questioning was used in 45% of the segments (Figure 9). Thirty-four percentage of the usage was high quality, 5% of the usage was mid quality, and 6% of the usage was low quality (Figure 12).

Mathematical questioning is when students ask questions that lead to generalization of a mathematical topic. Francisco and Maher report in their 2005 article, “Conditions for Promoting Reasoning in problem Solving: Insights from a Longitudinal Study,” that “students [need] to engage in well-defined, open-ended mathematical investigations as a context for the development of particular mathematical concepts and ways of reasoning” (Francisco & Maher, 2005, p. 370). In addition to appropriate tasks, students require that teachers mediate their thinking “by creating an environment that promotes mathematical understanding and problem solving through the questions they ask” (Mueller, Yankelewitz, & Maher, 2014, p. 317).
Students in Judith’s class conjectured the rules for solving equations through her thoughtful and probing questions. Students in Rose’s and Iris’ classes discovered patterns and generalizations about the patterns; again, both teachers guided student thinking by the questions they asked. It is important to note that the 45% usage of student questioning and reasoning consisted of student reasoning about the mathematics. The students did not ask any questions that led to a mathematical generalization. These types of questions might include for example, “Why does this rule work?” The students did not ask these types of questions.

**Common Core-Aligned Student Practices - Students Communicate About the Mathematics**

Although, this strategy identifies all student communication including one-word answers to teacher questions and procedural descriptions of algorithms, this type of communication is coded lower than when student communication is conceptual, e.g. the meaning of numbers related to a word problem, offering explanations, commenting on the reasoning of others, or discussing solution methods, etc. Students communicated in 85% of the segments (Figure 9). The quality of student communication was 45% high, 9% mid, and 31% low (Figure 12). Students were just as likely to respond with substantive statements such as explanations, describing a term or solution method, or commenting on the reasoning of others as they were to make brief one- or two-word comments. The difference between high, mid, and low student communication also distinguishes teacher styles and skills.

Judith had students explain their thinking throughout her lessons. Her questions followed up all student answers by asking them to explain their rationale. Maria’s
students reasoned and explained their mathematical choices as they completed their assignments. In several instances, teacher questioning was answered by right or wrong types of answers when the students did not elaborate how they arrived at their answer.

**Common Core-Aligned Student Practices - Task Cognitive Demand**

The goal of this strategy is to identify the students doing mathematical thinking. Task Cognitive Demand can be low if the task only requires students to follow procedures, fill in numbers in an equation or solve algorithms by rote. Task Cognitive Demand is high when problems have multiple solutions paths and students can identify and choose the solution path. In the 80 segments, Task Cognitive Demand was used 74% of the time (Figure 9). The quality of usage was high 44% of the time, mid 14% of the time, and low 16% of the time (Figure 12).

The cognitive demand of a task can be lowered by the teacher actions. Teachers can simplify the tasks by giving students the steps for finding the solution, thus lowering the cognitive demand of the task. An example of this was Tony’s eighth-grade lesson. Small groups of students were asked to figure out if there were sufficient ingredients to make a recipe. The students were making several attempts but were not heading in the right direction. Tony assessed the student struggles with a lack of understanding. He further explained the problem providing students with solution procedures students could use. In most instances teachers provided students step by step guidance. Although students were sometimes asked to explain their thinking; the teacher support lowered the task cognitive demand. According to Henningsen and Stein (1997, p. 546), it is the teacher’s responsibility to provide students with an appropriate amount of time for the
task and “scaffolding students consistently by pressing them to provide meaningful explanations or make meaningful connections” (p. 546).

**Common Core-Aligned Student Practices - Students Work with Contextualized**

A contextualized problem may include word problems, equations to represent situations, tables, or graphs, etc. This strategy also requires that students do the thinking for solving contextualized problems. It is possible for teachers to turn contextualized problems into procedural problems. Students were provided contextualized problems for solving problems and did the cognitive work for solving the problems in 63% of the segments (Figure 9). The quality of the segments was high in 36% of the segments, mid in 8% of the segments, with 18% of the segments low quality (Figure 12).

In segments about equivalent fractions, Deborah used a pan of brownies as a representation of equivalence. Tony gave students cylinders to figure out the volume. Rosita gave students rectangular prisms and tape measures to find the volume. All three teachers provided students with step-by-step instructions, thus turning the conceptualized problems into procedural problems.

Students tasks also carry a message about what mathematics means to the students. “The nature of the task can potentially influence and structure the way students think and can serve to limit or to broaden their views of the subject matter” (Henningsen & Stein, 1997, p. 252). Although most of the tasks presented to students in this study were conceptualized, some of the teachers provided explanations that were so complete that the tasks became procedural.
MQI Strategies Summary

The teachers in the study used the 13 positive MQI teaching strategies in most of the segments. Four strategies were used 80% or more of the time: Mathematical Language (90%), Student Contributions (85%), Mathematical Sense-Making (83%) and Explanations (80%). The least used strategies were Student Mathematical Questions and Reasoning (45%) and Multiple Procedures (43%). The three Error and Imprecision strategies, the one negative dimension, were used only in 3% of the segments. While student remediation was not observed frequently (51%), when students made mistakes the teacher acted immediately to correct errors. The quality of usage for all strategies tended to towards high quality rather than mid or low.

Results Analyzed by Grade Levels

Next, I consider the differences and similarities in the MQI dimensions and strategies by looking at grade levels. I divided the 80 segments into three grade levels: Grades 1 – 2, Grades 3 – 5, and Grades 6 – 8. Of the 80 segments, there were 33 segments in Grades 1 – 2, 26 segments in Grades 3 – 5, and 21 segments in Grades 6 – 8. There were five lessons observed in both Grades 1 – 2 and Grades 3 – 5. The difference in the number of segments was due to the length of each lesson. The Grades 1 – 2 lessons tended to be longer than the Grades 3 – 5 lessons. The Grade 6 – 8 lessons were also shorter, and I only observed four lessons at this level. The discussion follows a similar pattern as previous sections, first, a discussion of the three dimensions (except for Errors and Imprecisions due to the small number of incidences), then the thirteen strategies which comprise the three positive dimensions.
Comparisons of the Three Positive MQI Dimensions by Grades

When comparing the three positive MQI dimensions, all grade levels used strategies in the Richness of the Mathematics dimension more often than the other two dimensions—more than 50% of the time (Figure 14). The Richness of the Mathematics dimension also had the greatest difference between grades 1–2 and the other two grade levels (grades 1–2, 73%, grades 3–5 (54%) and grades 6–8 (55%)). The dimension of Working with Students and Mathematics showed all grades were within 10% points of each other. Grades 1–2 and 6–8 had about the same percentage of usage in Common Core-Aligned Student Practices (Figure 14).

![Dimension Usage by Grades](image)

*Figure 14. Dimension Usage by Grade.*

Comparison of the Three Positive MQI Dimensions by Grades and Quality of Usage

Teachers tended to use all three dimensions with a high quality of instruction rather than mid or low quality (Figure 15). However, the grades 1–2 teachers had a
greater amount of high quality instruction than the other two grade levels for all three dimensions. In Grades 3 – 5, all three of the dimensions had a quality of usage where the percentage of high-quality usage was about the same as low and mid usage combined. In Grades 6 – 8, the quality of usage was highest in the Richness of the Mathematics, (48%), and somewhat less high-quality usage in Working with Students and Mathematics (38%) and Common Core (43%, Figure 15).

Comparisons of the MQI Strategies by Grades

In this analysis, I look for similarities and differences in total usage. I review the usage of each MQI strategy by grade level and compare usage between strategies and between grades.
In the Richness of Mathematics dimension (Figure 16) there is a large difference between the grades in the use of Linking Between Representations and Explanations with teachers in the lower grades using these strategies more than the other grades. Mathematical Sense-Making is used by all the grades in about (80%) of the segments. Multiple Procedures or Solutions Methods is used by Grades 1 – 2 (52%), Grades 3 – 5 (46%) and Grades 6-8 (24%). Patterns and Generalizations strategies are used by all grades about 60% of the time. Mathematical Language is used by all three grade levels more than 80% of the time: 1 – 2 (94%), 3 – 4 (88%) and 6 – 8 (86%, Figure 16).

![Richness of the Mathematics Usage by Grade](image)

*Figure 16. Richness of the Mathematics Strategies Usage by Grade.*

In the Working with Students and Mathematics dimension (Figure 17), Remediation of Student Error and Difficulties is more often used by Grades 3 – 5 (65%) than the other grades (1 – 2 (45%), 6 – 8 (43%). Grades 3 – 5 teachers use Student
Figure 17. Working with Students and Mathematics Usage by Grade.

Mathematical Contributions in every segment (100%), grades 1 – 2 (82%), and grades 6 – 8 (90%, Figure 17). In the Common Core Aligned Student Practices dimension (Figure 18), students provided more explanations in Grades 6 – 8 (71%) than in the other grades. There was evidence of Student Mathematical Questioning and Reasoning in about half of the segments in grade 1 – 2 and 6 – 8 and less than half in grades 3 – 5. Student Communication was used the same in grades 1 – 2 (88%) and grades 3 – 5 (88%) and somewhat less in grades 6 – 8 (76%). Task Cognitive Demand was present in 76% of the 1 – 2-grade segments, 62% of the 3 – 5-grade segments and 86% of the 6 – 8-grade segments. The Students Work on Contextualized Problems was highest in grades 6 – 8.
(81%) and used less often in grades 3 – 5 (62%) and least in grades 1 – 2 (52%) as seen in Figure 18. Error! Reference source not found.

![Common Core-Aligned Student Practices Usage by Grade](image)

**Figure 18.** Common Core-Aligned Student Practices Usage by Grade.

In summary, overall, strategy usage was higher in grades 1 – 2. Notable exceptions were Remediation of Student Error and Teacher Uses Students Mathematical Contributions, which were higher in the middle grades. Also, Students Provide Explanations and Students Work with Contextualized Problems was used most often in the upper grades.

**Quality of MQI Code Usage by Grades**

This section provides a description of how well the teachers used the 13 MQI strategies divided by grade level. I take note of strategies whose usage was of high quality. In addition, I compare the quality of usage by grade level. In this comparison, I
present the chart first because the graphs are segmented by dimension so they can be seen clearly. The six Richness of Mathematics strategies provide a window into teacher actions that move instruction forward; as their actions relate to the mathematical content of the lesson.

The Grade 1 – 2 teachers had a high-quality usage of the Linking Between Representations strategy (70% high, Figure 19). The Grade 3 – 5 teachers had a high-quality usage of this strategy only half the time, while the Grades 6 – 8 teachers had high-quality usage less than a quarter of the time. The Grade 1 – 2 and Grades 6 – 8 teachers had a high quality of usage of the Explanation strategy 82% and 76% of the time respectively. Grades 3 – 5 used Explanations at a high quality about half the time (46%).

![Richness of Mathematics Strategies Quality by Grade-Part 1](image)

*Figure 19. Richness of the Mathematics Strategies Quality by Grade - Part 1.*

Sense-Making had a high quality of usage by Grades 1 – 2 teachers (70% high, and Grades 6 – 8 teachers (67% high) as seen in Figure 19. Multiple Methods or
Solutions had a high quality of usage in less than half the segments in all grades. The Patterns and Generalizations strategy was used at a high quality in Grades 1 – 2 and Grades 6 – 7 about half the time. In grades 3 – 5 a high quality of usage was observed in 8% of the segments. Mathematical Language was used at a high level in Grades 1 – 2 and 6 – 8 over half the time. The Grade 3 – 5 teachers used Mathematical Language at a high-quality level in 35% of the segments as shown in Figure 20.

![Richness of the Mathematics Strategies Quality by Grade-Part 2](image-url)

*Figure 20. Richness of the Quality of Mathematics Strategies Quality by Grade - Part 2.*

The next dimension to be discussed by grade level is Working with Students and Mathematics (Figure 21). This dimension has two strategies: Remediation of Student Error and Teacher Uses Student Mathematical Contributions. The Remediation of Student Error was not used often. The quality of usage was high in 33% of the segments for grades 6 – 8, 24% of the segments for grades 1 – 2, and 19% of the segments in grades 3 – 5. Teacher Uses Students Mathematical Contributions in 61% of the segments
for Grades 1 – 2 with a high quality. Fifty percent of the segments of grades 3 – 5 were high quality and 43% for Grades 6 – 8 of the segments were observed as high quality of instruction (Figure 21).

Figure 21. Working with Students and Math Quality by Grade.

The next dimension I describe for the quality of usage is the Common Core-Aligned Student Practices (Figure 22). The Students Provide Explanations strategy was observed at a high quality of usage in Grades 1 – 2 (58%), Grades 3 – 5 (31%) and 6 - 8 (43%). The quality of the Student Mathematical Questioning and Reasoning strategy was high in less than half the segments for all grades. The Students Communicate About the Mathematics strategy was of high quality in a little over half the segments for Grades 1 -2 (58%) and less than half of the segments in Grades 3 – 5 (35%), and 6 – 8 (38%). Task Cognitive Demand, was high for half the segments for Grades 1 -2 (52%) and 6 – 8 (52%). Grades 3 – 5 were observed as high in 27% of the segments. Contextualized
Problems was high in more than half the segments in Grades 6 – 8 (52%). Grades 1 – 2 and 3 – 5 had a high level of usage in about 30% of the segments (Figure 22).

Figure 22. Common Core-Aligned Student Practices Quality of Usage by Grade.

MQI Grade Usage Summary

Of the thirteen strategies (excluding Teacher Error and Imprecisions), Grades 1 – 2 usage was high quality (above 50%) in 9 of the 13 strategies (Figures 19 – 22). Grades 6 – 8 usage was high quality (above 50%) in 6 of the 13 strategies (Figures 19 – 22). Grades 3 – 5 usage was of high quality for only two strategies: Linking Between Representation (50%, Figure 19) and Teacher Uses Student Mathematical Contribution (50%, Figure 21).
Summary

The research question for this study was: “In what ways do elementary teachers of high achieving mathematics students deliver mathematics instruction.” The most obvious answer to this question is that the teachers in this study made few errors and provided strong Explanations, Sense-Making, Use of Mathematical Language, Linking between Representations and Student Communications. A summary and discussion of findings is in the final chapter.
CHAPTER 6

RESEARCH SUMMARY

Introduction

This study provides a detailed look inside the mathematics classrooms of eight teachers who scored at or above average on the MKT (Ball et al., 2005, p. 18) and who were teaching at a school within the Florida Conference where the NCE of a five-year average (2008 – 2012) on the IBTS (Hoover et al., 2007) was above the 50th NCE of the Florida Conference average NCE. The eight teachers in this study were from various grades: two first-grade teachers, a first- and second-grade teacher, a third-grade teacher, a fourth-grade teacher, a fifth-grade teacher, and two sixth through eighth-grade teachers.

I videotaped 14 class periods which were divided into 7.5-minute segments for analysis. The final segments in each lesson lasted no less than 3.5 minutes – for a total of 80 segments. Each segment was analyzed for the 16 MQI strategies (LMT Project, 2011).

Because teachers are the most important determiner of student learning, the strategies they use impacts student education (RAND Corporation, 2012). What these teachers do matters because “teachers near the top of the quality distribution got an entire year’s worth of additional learning out of their students compared to those near the bottom.” That is, a good teacher will get a gain of 1.5-grade level equivalent while a bad teacher will get 0.5 years during a single academic year” (Hanushek, 2014, p. 24).
Findings

1. The teachers used the three dimensions consistently: Working with Students and Mathematics (71%), Richness of the Mathematics (70%) and Common Core-Aligned Student Practices (66%) of the time.

2. There were few incidences of the strategies in the dimension Errors and Imprecisions (3%).

3. The teachers were more skillful using the strategies in the Richness of the Mathematics dimension (50% high quality) than Common Core-Aligned Student Practices (41% high quality) or Working with Students and Mathematics (39% high quality).

4. The three Richness of the Mathematics Strategies used the most were: Mathematical Language (90%), Mathematical Sense-making (83%) and Explanations (80%).

5. The Richness of the Mathematics Strategy used the least often was Multiple Procedures or Solutions Methods (43%).

6. The Working with Students and Mathematics Strategy used the most often was Teacher uses Student Mathematical Contributions (90%). This strategy was used with high quality 53% of the time.

7. The Common Core-Aligned Student Practices Strategy used the most often was Students Communicate about the Mathematics of the Segment (85%). This strategy was used with high quality 45% of the time.

8. All three grade levels used the Richness of Mathematics dimension more than the other two positive dimensions.
9. Grades 1 – 2 teachers used all six strategies in the Richness of Mathematics dimension more often than the other grades.

10. Grades 3 – 5 teachers used Mathematical Contributions more often than the other two grade clusters.

11. Grades 6 – 8 teachers used Task Cognitive Demand and Students Work with Contextualized Problems more often than the other grades.

Discussion of Major Findings and the Conceptual Framework

The MQI describes mathematical teaching through three relationships: how the teacher relates to the students, how the teacher relates to the mathematical content and what the teacher does to help students understand the material (LMT Project, 2011; LMT/Mathematics Instrument Development Group, n.d.; NCTE, 2012). The Instruction Triangle (Cohen et al., 2003; NCTE, 2012) is drawn with the teacher at the apex of the triangle, the students on the left and the content on the right side of the triangle, and bidirectional arrows connecting the teachers, students and content (Figure 23 Error! Reference source not found.).

Each dimension and code on the MQI provide a way to look at one of these relationships. Two of the dimensions: Richness of the Mathematics and Errors and Imprecisions, describe the relationship of the teacher to the content. The Working with Students and Mathematics dimension describes the relationship between the teacher and the students. The Common Core-Aligned Student Practices describe the relationship of the students to the content. All of this assumes that the Classroom Work is connected to Mathematics. In my study, only one segment of the 80 segments was not connected to mathematics.
When considering the Teacher – Content side of the triangle two things are obvious from my study: the teachers made very few errors, and the strategies in the Richness of the Mathematics dimension was used in 70% of the observed segments at a high-quality level 50% of the time. Teachers at all levels used Mathematical Sense-Making and Mathematical Language in more than 80% of the segments (Figure 7). This stands to reason because my sample of teachers scored at least average knowledge on the MKT test. These teachers know and understand the mathematics. They expect mathematics to make sense and they use appropriate mathematical language about 90% of the time (Figure 7). They also expect students to communicate consistently about the mathematics (Figure 9). The only Richness of the Mathematics strategy that was used
less than 50% of the time was Multiple Procedures or Solutions Methods (43%). The quality of Multiple Procedures or Solutions Methods was 25% high.

Another side of the triangle was the relationship of the teacher to the student. How does the teacher connect with the student to make the mathematics understandable? The teachers made a great effort to use the students’ contributions (100% of the time for grades 3 – 5). It’s clear to me that the students made few errors because of this, plus the fact that the teachers explained everything so well and expected the students to make sense of the mathematics. It seems to me that the low incidence of remediation is related to the fact that the teachers have a strong command of the mathematics and the pedagogy to teach math. As I watched these teachers, I particularly noted that they were very careful to provide explanations, questions, and activities that carefully guided students to understanding. I believe the students needed less remediation because of the teachers’ knowledge and use of various strategies or the teachers’ choice of problems.

The Common Core-Aligned Student Practices dimension relates to the relationship between the students and the content side of the triangle. Comparing the Richness of the Mathematics dimension with the Common Core-Aligned Student Practices dimension, it is easy to see the teachers often used strategies that allowed them and their students to depend on the teachers’ knowledge rather than the student connection to the content. These teachers are working hard to see that students understand mathematics. Many segments showed teachers providing students step-by-step explanations. Most of the teachers moved students from one idea to the next with detailed explanations. Even when the teachers were allowing students to make contributions to the ideas, they guided the discussion through their questions. The
teachers either poured the information into the students or they drew it out of them through their examples or their questioning. There were very few instances where students could ponder, mull over, or feel anxiety about solving a problem.

In the observed segments, the students were given fewer opportunities to connect with the mathematics. The Common Core Practice Standards (National Governors Association Center for Best Practices Council of Chief State School Officers, 2010b) describe students as “practitioners of the discipline of mathematics increasingly…to engage with the subject matter as they grow in mathematical maturity and expertise” (p. 8). Of the Common Core-Aligned Student Practices, Student Mathematical Questioning and Reasoning (45%, Figure 9) was used the least. Of the 13 strategies, only Multiple Procedures or Solution Methods (43%, Figure 7) was used less. In the Common Core-Aligned Student Practices (Figure 12) the high quality of usage in all five of the strategies was below 50%. “Past research shows that students of all levels face serious difficulties acquiring competency in reasoning and developing proofs. This research also shows that teachers, especially elementary teachers, have weak content knowledge of reasoning and proving” (Stylianides & Stylianides, 2006, p. 202). My study seems to confirm that research. Although the teachers know how to impart knowledge to the student, they are not adept in providing opportunities for students to actively do the reasoning and proving for themselves.

It appears that most of these classrooms are places where students made very few errors. Therefore, the usage of the Remediation of Student Errors and Difficulties was found in only 51% of the segments (Figure 8). Remediation of Student Errors and Difficulties was used at a high quality 25% of the time. When errors were spotted, the
teacher swooped in and cleared them away. When Rose spotted several student errors in representing three addends, she quickly adjusted her plans, bringing the students back to the carpet. Although it is commendable to be that receptive toward student needs, real-world mathematics problems require perseverance which cannot be developed without a struggle. In general, the teachers in this study did not give students time to persevere and understand the given problems on their own.

Dividing material into small chunks and pre-chewing the contents for students did not allow them to make mistakes. I think the students in these classrooms seem to understand the material well because they have been given a very clear blueprint to follow, so they do not make many mistakes. In some segments, it is not clear if the students understand the material or have just memorized teachers’ rationalizations and procedures. It might be possible to explain why teachers made these choices, since all the teachers in the study scored above average on the MKT, and its questions identify knowledge of the content and the pedagogy. The teachers in this study have knowledge of the content and the pedagogy needed to provide student learning and support.

It is also possible to explain these results by comparing them to the results found in the Methods and Findings from an Exploratory Research Project on Eight-Grade Mathematics Instruction in Germany, Japan, and United States (Stigler, Gonzales, Kawanaka, Knoll, & Serrano, 1999). Stigler, et al (1999) found that in the United States lessons:

. . . followed what might be called an acquisition/application script. During the acquisition phase, students are expected to learn how to solve particular types of problems often through watching demonstrations by the teacher or their peers.

During the application phase, students are expected to practice what they have learned. Lessons in Japan look quite different. They follow what might be called
a ‘problematizing script. In this script, problem-solving becomes the context in which competencies are simultaneously developed and utilized. The goal of the lesson becomes to understand math principles rather than getting the correct answer to a mathematical problem.

The lesson starts with seat work; then the teachers provide instruction. In the Japanese lesson, the order is reversed. It is expected that student will struggle because they have not already acquired a procedure to solve the problem. The time spent struggling on one’s own to work out a solution is considered an important part of the lesson. (Stigler et al., 1999, p. 135)

The length of class periods may also affect the instructional strategies observed during observations. When comparing individual teachers, the teachers with the highest usage of Common Core-Aligned Practices also had the longest lessons. It is possible that when teachers have more time, they may be able to give students time to process the content on their own.

The one notable exception was Judith’s classroom. She provided space in the lessons for the student to truly figure out why numbers work the way they do. Her lessons seemed to revolve around the idea that to fully understand the concept; they must resolve some mental equilibrium. This strategy produced almost palpable physical pain among the students. But when she compared the game to keeping equations balanced when solving for the unknown, the students were primed and invested in keeping the expressions balanced. In another lesson, she proceeded to ask the students to prove the correct placement of the fractions. She gave students chances to explain, dispute ideas, change their minds, and summarize what others said. She was the “guide on the side” providing students with just enough information to keep them focused on her learning objectives.

Comparing the instruction observed in this study, I made the following observations:
• The teachers made few errors and used precise language in instruction.

• There was high usage and quality of strategies connected with the teacher and the content of the Instructional Triangle (Figure 23 Error! Reference source not found.).

• There was consistent usage and partial high quality of strategies connected with the teacher and the student side of the Instructional Triangle (Figure 23 Error! Reference source not found.).

• There was low usage and inconsistent quality of the strategies on the student and the content side of the Instructional Triangle (Figure 23 Error! Reference source not found.).

These observations could be caused by teacher knowledge, allotted class time, teacher beliefs this it is their responsibility to provide clear and careful instruction. This study does not attempt to identify the causes, but only describe the instructional strategies used by the study teachers.

These findings are broadly in harmony with previous research such as Ball and Bass’ (2000) research of specialized teacher knowledge. Study teachers made mistakes in content, notation, or clarity of explanation in only 3% of the segments. In the 80 segments, the study teachers consistently used explanations, mathematical sense-making, linking between representations, and the use of mathematical language during instruction.

According to Anthony and Walshaw (2009), effective teachers respond “toward the needs of all their students” (p. 157). Study teachers used student contributions to move instruction forward. They consistently addressed student difficulties and provided conceptual remediation.
Hanushek and Rivkin (2006) and Pang (2009) have described the characteristics of effective teaching as the relationship between the teacher’s actions and the students’ ability to construct mathematical models. The study teachers provided students with explanations that allowed students to understand and build conceptual models of the concepts taught during lessons observed.

These teachers have strong knowledge and skill on the teacher to content side of the Instructional Triangle (Figure 23). They know the content. They know how to explain the math. They provide students with explanations that are meaningful not just procedural. Although the Richness of the Mathematics dimension of the MQI was a teacher strength, they did not use multiple solutions or patterns and generalizations as often as other strategies in this dimension.

The weakness of the teachers in this study was in the student-content relationship side of the Instructional Triangle. This side of the Instructional Triangle describes the students’ use of critical thinking to solve math problems. “Critical Thinking requires effort to collect, interpret, analyze and evaluate information to arrive at a reliable and valid conclusion. In teaching Mathematics in schools, Critical Thinking needs to be integrated and emphasized in the curriculum so that student can learn the skills and apply it to improve their performance and reasoning ability” (Chukwuyenum, 2013, p. 24). The teachers in this study were doing a lot of the critical thinking for the students. Although both students and teachers have lower usage and quality in the strategies that provide for critical thinking, the teachers are doing more critical thinking than students.

The instructional delivery in most of the classrooms in this study could be considered as “sage on the stage.” The teachers provide students with the information
needed to solve mathematical problems. They ask leading questions to guide student thinking about the mathematical situation. Most of the time the students in these classrooms passively receive instruction either through teacher talk or teacher questions.

**Implications**

This study appears to support a change in teacher application of instructional strategies that increase student ability to think mathematically. According to NCTM, in order for students to effectively think mathematically, they must be able to solve problems, reason, communicate, make connections, and understand and use explanations (NCTM, 2014). Common Core Practices include making sense of problems, reasoning abstractly and quantitatively, constructing and critiquing mathematical reasoning, modeling with mathematics, choosing correct mathematical tools to solve problems, attend to precision, and find patterns (National Governors Association Center for Best Practices Council of Chief State School Officers, 2010c). Deborah Ball describes the goal of teaching and learning mathematics is for students to become people who are fluent and skillful with mathematics and see mathematics as interesting and useful (Ball, 2017). Teacher ability to more effectively apply Common Core-Aligned Practices found in the MQI may improve student ability and allow them to become fluent and skillful with mathematics in all concepts. One must ask, “What happens to students who aren’t consistently encouraged to find multiple solutions? What happens to students who expect the teacher to explain and provide procedures?”

**Limitations**

My analysis has concentrated on effective teachers in the Florida Conference of Seventh-day Adventists and their use of effective teaching strategies through the lenses of
the MQI. Some may argue that there are other ways of assessing teachers that were not used such as classroom artifacts, lesson plans and student work, or use of other observation tools. Granting that there are other methods of assessing teacher effectiveness, the MQI provides a comprehensive way of describing what effective mathematics instruction looks like. It allows for readers to picture effective instruction and possibly see their own practices in the study teachers.

**Recommendations**

Based on the findings of this study, the following recommendations are suggested.

**Florida Conference of Seventh-day Adventists**

I recommend that all mathematics teachers be provided intensive professional development in mathematics including the MQI Strategies and Standards for Mathematical Practice Common Core State Standards.

**Practitioners**

I recommend that math teachers:

- Understand the requirements of the Common Core Mathematics Standards Practices. This will help them understand the need for change.
- Become familiar with the MQI rubric and receive training using the MQI rubric to improve current practices.
- Receive professional development that enhances the skills needed to use the Common Core-Aligned Student Practices to solve grade level problems.
• Provide students with direct instruction on how to think critically, i.e. transferring information to a new context and generalizing.

• Administrators

• I recommend that superintendents, building administrators, and instructional coaches:

  • Adopt the Common Core Mathematics Standards Practices. These practices will allow student to become mathematical thinkers.

  • Receive professional development in using the MQI rubric to improve mathematics instruction, through observation and feedback to teachers.

  • Provide teachers with training using the MQI rubric and feedback as teachers develop their math skills. If provided with instruction and feedback the teachers would better understand not only the Common Core Math Standards, but also the Common Core Mathematics Practices.

Researchers

I recommend additional research in the following areas:

• How do teacher’s beliefs influence their use of MQI strategies?

• How does this sample of teachers compare with a random sample of teachers in the Adventist system?

In addition to the instructional strategies identified on the MQI, are there other instructional strategies that teachers could use to improve student comprehension of mathematics concepts?
Final Thoughts

Undertaking this research has been an invaluable experience. I have gained a deeper understanding of the research process. I have learned that research, like a mystery novel, is a living and breathing artifact like the characters in a novel.

This research study has also enriched my practice as a math teacher. Math is not an immoveable list of rules but a habit of mind, a way of looking at the world. The instructional strategies a teacher uses can either allow students to be spectators or mathematicians. A mathematician finds problems and decides the problem goals, chooses representations, chooses and implements solution procedures, and compares results with goals. Math instruction should allow for students to flexibly use numbers by composing and decomposing them. Mathematicians understand and use concrete and abstract ways to represent concepts. Teachers need to have these skills themselves, they need to understand how these skills are taught, and they urgently need to allow students to think for themselves. Ellen G. White (1903) counsels that “it is the work of true education to develop this power, [the power to think and do] to train the youth to be thinkers, and not mere reflectors of other men’s thought” (p. 17). Mathematician, author, and Dartmouth Mathematics Department Chair, John Wesley Young, identifies how we should teach mathematics.

The chief end of mathematical study must be to make the students think. If mathematical teaching fails to do this, it fails altogether. The mere memorizing of a demonstration in geometry has about the same educational value as the memorizing of a page from the city directory. (Young, 1911, p. 4)

If we are to improve mathematics achievement, as educators in the 21st century, we must teach so that our students think mathematically. Our students should see the grace of mathematics, the beauty of mathematics, ask interesting questions about
mathematics, be able to flexibly model the world and abstract its patterns with mathematics. Our change in focus may not only help students love mathematics, but it may also improve our scores on international assessment.
APPENDIX A

CORRESPONDENCE FROM FRANK RUNNELS

February 18, 2014

Institutional Review Board
Andrews University
4150 Administrative Drive, Room 322
Berrien Springs, MI 49104-0355
irb@andrews.edu

To the Andrews University Institutional Review Board:

I hereby grant Anna Adkins permission to conduct doctoral research titled, “Instructional Strategies and Beliefs of Teachers Whose Students Are in the Top Half on the ITBS in the Florida Conference of Seventh-day Adventists: A Qualitative Study” in the Florida Conference of Seventh-day Adventists.

After IRB approval, I understand that she will contact the lead teacher and/or principal of the following eligible schools: Avon Park, Daytona Beach, Longwood, Okeechobee, Port Charlotte and St. Petersburg. Following this committee’s recommendation, Anna will get permission from the lead teacher or principal to contact the teachers. Once the school’s permission has been granted she will contact eligible teachers. If you have any further questions do not hesitate to contact me.

Sincerely,

Frank Runnels
Superintendent of Education
Florida Conference of Seventh-day Adventists

FR/cg
APPENDIX B

IRB APPROVAL

February 7, 2014

Anna Adkins
Tel: (407) 765-1251
Email: anadns@gmail.com

RE: APPLICATION FOR APPROVAL OF RESEARCH INVOLVING HUMAN SUBJECTS
IRB Protocol #: 14-021 Application Type: Original Dept.: Leadership
Review Category: Expedited Action Taken: Approved Advisor: Shirley Freed
Title: Instructional strategies and beliefs of teachers whose students are in the top half on the ITBS in the Florida Conference of Seventh-day Adventists: A qualitative study

This letter is to advise you that the Institutional Review Board (IRB) has reviewed and approved your IRB application of research involving human subjects entitled: Instructional strategies and beliefs of teachers whose students are in the top half on the ITBS in the Florida Conference of Seventh-day Adventists: A qualitative study” IRB protocol number 14-021 under Expedited category. This approval is valid until March 7, 2015. If your research is not completed by the end of this period you must apply for an extension at least four weeks prior to the expiration date. We ask that you inform IRB whenever you complete your research. Please reference the protocol number in future correspondence regarding this study.

Any future changes made to the study design and/or consent form require prior approval from the IRB before such changes can be implemented. Please use the attached report form to request for modifications, extension and completion of your study.

While there appears to be no more than minimum risk with your study, should an incidence occur that results in a research-related adverse reaction and/or physical injury, this must be reported immediately in writing to the IRB. Any project-related physical injury must also be reported immediately to the University physician, Dr. Rechert, by calling (269) 473-2222. Please feel free to contact our office if you have questions.

Best wishes in your research.

Sincerely,

Mordecai Ong
Research Integrity & Compliance Officer

Institutional Review Board - 4150 Administration Dr Room 223 - Berrien Springs, MI 49104-4356
Tel: (269) 471-6361 Fax: (269) 471-6543 E-mail: irb@andrews.edu
REFERENCE LIST
REFERENCE LIST


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CURRICULUM VITA
CAREER PROFILE

2015-Present  Teacher, Forest City Adventist School, Altamonte Springs, FL
2010-2015  Vice Principal, Orlando Junior Academy, Orlando, FL
2010 Winter  Adjunct Professor, Southern Adventist University, Collegedale, TN
2005-2010  Special Education Director, Forest Lake Education Center, Longwood, FL
2001-2005  Ladies Dean, Garden State Academy, Tranquility, NJ
1986-2000  Teacher 3 & 4, Waldwick Junior Academy, Waldwick, NJ
1984-1986  Grades 1-4, Central Jersey SDA School, Perth Amboy, NJ
1982-1984  Title I Reading Teacher, Robert Kennedy Action Corp., Lancaster, MA
1981-1982  Teacher Grades 5-7, Meadowbrook SDA School, Hudson, MA
1979-1982  Teacher Grades 1-3, Lakewood SDA School, Lakewood, OH

EDUCATION

2018  PhD Leadership  Andrews University, Berrien Springs, MI
1984  MEd in Special Education  Fitchburg State College, Fitchburg, MA
1979  BS in Elementary Education  Atlantic Union College, South Lancaster, MA

PROFESSIONAL ACTIVITIES

2007- Present  Committee Member  Florida Conference Special Needs Committee, Winter Springs, FL
April 2013  Committee Member  Osceola Adventist Christian School, Kissimmee, FL

WORKSHOPS & PRESENTATION

June 2014  Implementing New Math Standards Using “Go Math”, Florida Conference
October 2013  Exploring the Possibilities of Science, Florida Conference
August 2012  Creating an Edible Schoolyard, NAD National Teachers Convention
August 2012  Leading Mathematical Discussion, NAD National Teachers Convention