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Joon Hyuk Kang
Andrews University, kang@andrews.edu

Timothy Robertson
Andrews University, robertsont@andrews.edu

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REGION OF SMOOTH FUNCTIONS FOR POSITIVE SOLUTIONS TO AN ELLIPTIC BIOLOGICAL MODEL

Timothy Robertson[1], Joon H. Kang[2 §]
[1,2]Department of Mathematics
Andrews University
Berrien Springs, MI. 49104, USA

Abstract: The non-existence and existence of the positive solution to the generalized elliptic model

$$\begin{aligned} &\Delta u + g(u,v) = 0 \ \text{ in } \ \Omega, \\ &\Delta v + h(u,v) = 0 \ \text{ in } \ \Omega, \\ &u = v = 0 \ \text{ on } \ \partial\Omega, \end{aligned}$$

were investigated.

Key Words: non-existence and existence of the solution, positive solution, generalized elliptic model

1. Introduction

The question in this paper concerns the existence of positive coexistence states when all growth rates are nonlinear and combined, more precisely, the existence of the positive steady state of

$$\begin{aligned} &\Delta u + g(u,v) = 0 \ \text{ in } \ \Omega, \\ &\Delta v + h(u,v) = 0 \ \text{ in } \ \Omega, \\ &u = v = 0 \ \text{ on } \ \partial\Omega, \end{aligned}$$

where Ω is a bounded domain in R^N with smooth boundary $\partial\Omega$, and $g, h \in C^2$ are such that $g_{uu} < 0, h_{vv} < 0, g_{uv} > 0, h_{uv} > 0$.

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§Correspondence author

2. Preliminaries

In this section, we state some preliminary results which will be useful for our later arguments.

Definition 2.1. (upper and lower solutions)

$$\begin{cases} \Delta u + f(x,u) = 0 \ \text{ in } \ \Omega, \\ u|_{\partial\Omega} = 0 \end{cases} \tag{1}$$

where $f \in C^{\alpha}(\bar{\Omega} \times R)$ and Ω is a bounded domain in R^n.

(A) A function $\bar{u} \in C^{2,\alpha}(\bar{\Omega})$ satisfying

$$\begin{cases} \Delta \bar{u} + f(x,\bar{u}) \le 0 \ \text{ in } \ \Omega, \\ \bar{u}|_{\partial\Omega} \ge 0 \end{cases}$$

is called an upper solution to (1).

(B) A function $\underline{u} \in C^{2,\alpha}(\bar{\Omega})$ satisfying

$$\begin{cases} \Delta \underline{u} + f(x,\underline{u}) \ge 0 \ \text{ in } \ \Omega, \\ \underline{u}|_{\partial\Omega} \le 0 \end{cases}$$

is called a lower solution to (1).

Lemma 2.1. *Let $f(x,\xi) \in C^{\alpha}(\bar{\Omega} \times R)$ and let $\bar{u}, \underline{u} \in C^{2,\alpha}(\bar{\Omega})$ be respectively, upper and lower solutions to (1) which satisfy $\underline{u}(x) \le \bar{u}(x), x \in \bar{\Omega}$. Then (1) has a solution $u \in C^{2,\alpha}(\bar{\Omega})$ with $\underline{u}(x) \le u(x) \le \bar{u}(x), x \in \bar{\Omega}$.*

We also need some information on the solutions of the following logistic equations.

Lemma 2.2.

$$\begin{cases} \Delta u + uf(u) = 0 \ \text{ in } \ \Omega, \\ u|_{\partial\Omega} = 0, u > 0, \end{cases}$$

where f is a decreasing C^1 function such that there exists $c_0 > 0$ such that $f(u) \le 0$ for $u \ge c_0$ and Ω is a bounded domain in R^n.

If $f(0) > \lambda_1$, then the above equation has a unique positive solution. We denote this unique positive solution as θ_f.

The main property about this positive solution is that θ_f is increasing as f is increasing.

3. Existence and Nonexistence of Steady State

We consider

$$\begin{aligned} &\Delta u + g(u,v) = 0 \ \text{ in } \ \Omega \\ &\Delta v + h(u,v) = 0 \ \text{ in } \ \Omega \\ &u = v = 0 \ \text{ on } \ \partial\Omega, \end{aligned} \tag{2}$$

where Ω is a bounded domain in R^N with smooth boundary $\partial\Omega$ and $g, h \in C^2$ are such that $g_{uu} < 0, h_{vv} < 0, g_{uv} > 0, h_{uv} > 0,\ g(0,v) \geq 0, h(0,v) \geq 0$.

We derive the following nonexistence result, which establishes a necessary condition for the existence of a positive solution to (2).

Theorem 3.1. *Suppose $g_u(0,0) > \lambda_1, h_v(0,0) > \lambda_1$, where λ_1 is the first eigenvalue of $-\Delta$ with homogeneous boundary condition, and there is $c_0 > 0$ such that $g_u(u,0) < 0$ and $h_v(0,v) < 0$ for $u > c_0, v > c_0$.*

(1) If $g_u(0,0) \geq h_v(0,0), -1 \leq g_{uu} < 0, h_{vv} \leq -1$ and

$$\inf(h_{uv})\inf(g_{uv}) + \inf(h_{uv}) + \inf(h_{vv})\sup(h_{uv}) + \inf(h_{vv}) \geq 0,$$

then (2) has no positive solution.

(2) If $g_u(0,0) \leq h_v(0,0), -1 \leq h_{vv} < 0, g_{uu} \leq -1$ and

$$\inf(g_{uv})\inf(h_{uv}) + \inf(g_{uv}) + \inf(g_{uu})\sup(g_{uv}) + \inf(g_{uu}) \geq 0,$$

then (2) has no positive solution.

Proof. Suppose the conditions in (1) or (2) holds and (2) has a positive solution (u,v).

By the Mean Value Theorem, there is $\bar{u}$ such that $0 \leq \bar{u} \leq u$ and $g(u,v) - g(0,v) = ug_u(\bar{u},v)$, and so by the monotonicity of g_u

$$\begin{aligned} \Delta u + ug_u(u,v) \leq& \Delta u + ug_u(\bar{u},v) \\ =& \Delta u + g(u,v) - g(0,v) \\ =& \Delta u + g(u,v) \\ =& 0. \end{aligned}$$

Similarly, we can prove that

$$\Delta v + vh_v(u,v) \leq 0.$$

Hence, (u,v) is an upper solution to

$$\begin{aligned} &\Delta u + ug_u(u,v) = 0 \ \text{ in } \ \Omega \\ &\Delta v + vh_v(u,v) = 0 \ \text{ in } \ \Omega \\ &u = v = 0 \ \text{ on } \ \partial\Omega. \end{aligned}$$

By the conditions $(\tilde{u}, \tilde{v}) = (\theta_{g_u(\cdot,0)}, \theta_{h_v(0,\cdot)})$ exist. We claim that for sufficiently small $\epsilon > 0$, $(\epsilon\tilde{u}, \epsilon\tilde{v})$ is a lower solution to

$$\begin{aligned}&\Delta u + u g_u(u,v) = 0 \ \text{ in } \ \Omega\\&\Delta v + v h_v(u,v) = 0 \ \text{ in } \ \Omega\\&u = v = 0 \ \text{ on } \ \partial\Omega.\end{aligned}$$

By the monotonicity of g_u, we have

$$\begin{aligned}\Delta(\epsilon\tilde{u}) + \epsilon\tilde{u} g_u(\epsilon\tilde{u}, \epsilon\tilde{v}) &\geq \Delta(\epsilon\tilde{u}) + \epsilon\tilde{u} g_u(\tilde{u}, 0)\\&= \epsilon[\Delta(\tilde{u}) + \tilde{u} g_u(\tilde{u}, 0)]\\&= 0.\end{aligned}$$

Similarly, we can prove that

$$\Delta(\epsilon\tilde{u}) + \epsilon\tilde{u} g_u(\epsilon\tilde{u}, \epsilon\tilde{v}) \geq 0.$$

Hence, we conclude that $(\epsilon\tilde{u}, \epsilon\tilde{v})$ is a lower solution to

$$\begin{aligned}&\Delta u + u g_u(u,v) = 0 \ \text{ in } \ \Omega\\&\Delta v + v h_v(u,v) = 0 \ \text{ in } \ \Omega\\&u = v = 0 \ \text{ on } \ \partial\Omega.\end{aligned}$$

Therefore, by the Lemma 2.1, there is a positive solution to

$$\begin{aligned}&\Delta u + u g_u(u,v) = 0 \ \text{ in } \ \Omega\\&\Delta v + v h_v(u,v) = 0 \ \text{ in } \ \Omega\\&u = v = 0 \ \text{ on } \ \partial\Omega,\end{aligned}$$

which contradicts to the result in [1]. We now establish a sufficient condition for existence of a positive solution to (2).

Theorem 3.2. *Suppose $g_u(0,0) > \lambda_1, h_v(0,0) > \lambda_1$, and there are $M > 0, N > 0$ such that $g(M,N) < 0, h(M,N) < 0$.*

Then there is a positive solution to (2).

Proof. By the condition, we have an upper solution (M, N) to (2). Let ϕ be the first eigenfunction of $-\Delta$ with homogeneous boundary condition. Then, by the continuity of g_u and h_v and the assumption that $g_u(0,0) > \lambda_1, h_v(0,0) > \lambda_1$, $g_u(\epsilon\phi, \epsilon\phi) > \lambda_1$ and $h_v(\epsilon\phi, \epsilon\phi) > \lambda_1$ for sufficiently small $\epsilon > 0$.

By the Mean Value Theorem, there are $\tilde{u}, \tilde{v}$ such that $0 \leq \tilde{u} \leq \epsilon\phi, 0 \leq \tilde{v} \leq \epsilon\phi$ and

$$\begin{aligned}&g(\epsilon\phi, \epsilon\phi) - g(0, \epsilon\phi) = \epsilon\phi g_u(\tilde{u}, \epsilon\phi)\\&h(\epsilon\phi, \epsilon\phi) - h(\epsilon\phi, 0) = \epsilon\phi h_v(\epsilon\phi, \tilde{v}).\end{aligned}$$

Hence, by the monotonicity of g_u and h_v,

$$\begin{aligned}\Delta(\epsilon\phi) + g(\epsilon\phi,\epsilon\phi) \geq& \Delta(\epsilon\phi) + g(\epsilon\phi,\epsilon\phi) - g(0,\epsilon\phi)\\ =& \Delta(\epsilon\phi) + \epsilon\phi g_u(\tilde{u},\epsilon\phi)\\ \geq& \epsilon(-\lambda_1\phi) + \epsilon\phi g_u(\epsilon\phi,\epsilon\phi)\\ =& \epsilon\phi[-\lambda_1 + g_u(\epsilon\phi,\epsilon\phi)]\\ >& 0,\end{aligned}$$

and

$$\begin{aligned}\Delta(\epsilon\phi) + h(\epsilon\phi,\epsilon\phi) \geq& \Delta(\epsilon\phi) + h(\epsilon\phi,\epsilon\phi) - h(\epsilon\phi,0)\\ =& \Delta(\epsilon\phi) + \epsilon\phi h_v(\epsilon\phi,\tilde{v})\\ \geq& \epsilon(-\lambda_1\phi) + \epsilon\phi h_v(\epsilon\phi,\epsilon\phi)\\ =& \epsilon\phi[-\lambda_1 + h_v(\epsilon\phi,\epsilon\phi)]\\ >& 0.\end{aligned}$$

Hence, $(\epsilon\phi,\epsilon\phi)$ is a lower solution to (2). Therefore, by the Lemma 2.1, there is a positive solution to (2).

4. Existence Region for Steady State

We consider

$$\begin{aligned}&\Delta u + g(u,v) = 0 \ \text{ in } \ \Omega\\ &\Delta v + h(u,v) = 0 \ \text{ in } \ \Omega\\ &u = v = 0 \ \text{ on } \ \partial\Omega,\end{aligned} \tag{3}$$

where Ω is a bounded domain in R^N with smooth boundary $\partial\Omega$ and $g,h \in C^2$.

We prove the following existence results.

Theorem 4.1. *Suppose $g_u(0,0) > \lambda_1, g(0,v) \geq 0, g_{uu} < 0, g_{uv} > 0$ and there is $c_0 > 0$ such that $g_u(u,0) < 0, g(u,v) < 0$ for $u > c_0, v > c_0$.[$h_v(0,0) > \lambda_1, h(u,0) \geq 0, h_{vv} < 0, h_{uv} > 0$ and there is $c_0 > 0$ such that $h_v(0,v) < 0, h(u,v) < 0$ for $u > c_0, v > c_0$.] Then there is a number $M(g) < \lambda_1$ [$N(h) < \lambda_1$] such that for any $h \in C^2$ such that $h(u,0) \geq 0, h_{uv} > 0, h_{vv} < 0$, $h_v(0,v) < 0, h(u,v) < 0$ for $u > c_0, v > c_0$ and $h_v(0,0) > M(g)$[for any $g \in C^2$ such that $g_{uu} < 0, g_{uv} > 0$, $g_u(u,0) < 0, g(u,v) < 0$ for $u > c_0, v > c_0$, and $g_u(0,0) > N(h)$], (3) has a positive solution u^+, v^+ in Ω.*

Proof. Let $\underline{u} = \theta_{g_u(\cdot,0)}$ be the unique positive solution to

$$\begin{aligned}\Delta u + u g_u(u,0) &= 0 \ \text{ in } \ \Omega\\ u &= 0 \ \text{ on } \ \partial\Omega.\end{aligned}$$

Let $M(g) = \lambda_1(-h_v(\theta_{g_u(\cdot,0)}, 0))$ be the smallest eigenvalue of

$$\begin{aligned} -\Delta Z - (h_v(\theta_{g_u(\cdot,0)}, 0) - h_v(0,0))Z &= \mu Z \text{ in } \Omega \\ Z &= 0 \text{ on } \partial\Omega. \end{aligned}$$

and $\omega_0(x)$ be the corresponding normalized positive eigenfunction. By the monotonicity, $M(g) < \lambda_1$. Let $\underline{v} = \epsilon\omega_0(x)$. Let $h \in C^2$ be such that $h_{uv} > 0, h_{vv} < 0$, $h_v(0,v) < 0, h(u,v) < 0$ for $u > c_0, v > c_0$ and $h_v(0,0) > M(g)$. Then, by the Mean Value Theorem, there is $\tilde{u}$ and $\tilde{v}$ such that

$$\begin{aligned} &0 \le \tilde{u} \le \underline{u} \\ &0 \le \tilde{v} \le \underline{v} \\ &g(\underline{u}, \underline{v}) - g(0, \underline{v}) = \underline{u} g_u(\tilde{u}, \underline{v}) \\ &h(\underline{u}, \underline{v}) - h(\underline{v}, 0) = \underline{v} h_u(\underline{u}, \tilde{v}), \end{aligned}$$

so by the monotonicity of g_u and h_v, for sufficiently small $\epsilon > 0$,

$$\begin{aligned} \Delta\underline{u} + g(\underline{u}, \underline{v}) \ge& \Delta\underline{u} + g(\underline{u}, \underline{v}) - g(0, \underline{v}) \\ =& \Delta\underline{u} + \underline{u} g_u(\tilde{u}, \underline{v}) \\ \ge& \Delta\underline{u} + \underline{u} g_u(\underline{u}, \underline{v}) \\ =& \Delta\underline{u} + \underline{u}[g_u(\underline{u}, 0) + g_u(\underline{u}, \underline{v}) - g_u(\underline{u}, 0)] \\ =& \underline{u}[g_u(\underline{u}, \underline{v}) - g_u(\underline{u}, 0)] \\ >& 0 \text{ in } \Omega \end{aligned}$$

and

$$\begin{aligned} \Delta\underline{v} + h(\underline{u}, \underline{v}) \ge& \Delta\underline{v} + h(\underline{u}, \underline{v}) - h(\underline{u}, 0) \\ =& \Delta\underline{v} + \underline{v} h_v(\underline{u}, \tilde{v}) \\ \ge& \Delta\underline{v} + \underline{v} h_v(\underline{u}), \underline{v}) \\ =& \Delta(\epsilon\omega_0) + \epsilon\omega_0 h_v(\theta_{g_u(\cdot,0)}, \epsilon\omega_0) \\ =& \Delta(\epsilon\omega_0) + \epsilon\omega_0[h_v(\theta_{g_u(\cdot,0)}, 0) + h_v(\theta_{g_u(\cdot,0)}, \epsilon\omega_0) - h_v(\theta_{g_u(\cdot,0)}, 0)] \\ =& \epsilon[h_v(0,0)\omega_0 - M(g)\omega_0] + \epsilon\omega_0[h_v(\theta_{g_u(\cdot,0)}, \epsilon\omega_0) - h_v(\theta_{g_u(\cdot,0)}, 0)] \\ \ge& \epsilon\omega_0[h_v(0,0) - M(g)] + \epsilon^2\omega_0^2 \inf(h_{vv}) \\ >& 0 \text{ in } \Omega. \end{aligned}$$

So, $\underline{u} > 0, \underline{v} > 0$ is a lower solution to (3). But, by the condition, there is a sufficiently large upper solution to (3). Therefore, there is a positive solution u^+, v^+ of (3).

For the next Theorem, we set

$$S_g = \{h \in C^2 | h_{uv} > 0, M \le h_{vv} < 0, h(u,0) \ge 0,$$

there is $c_0 > 0$ such that $h(u,v) < 0$ for $u > c_0, v > c_0\}$ for $g \in C^2$ such that $g_{uu} < 0, g_{uv} > 0, g(0,v) \ge 0$, there is $c_0 > 0$ such that $g(u,v) < 0$ for $u > c_0, v > c_0$ and

$$S_h = \{g \in C^2 | N \le g_{uu} < 0, g_{uv} > 0, g(0,v) \ge 0,$$

there is $c_0 > 0$ such that $g(u,v) < 0$ for $u > c_0, v > c_0\}$ for $h \in C^2$ such that $h_{uv} > 0, h_{vv} < 0, h(u,0) \ge 0$,there is $c_0 > 0$ such that $h(u,v) < 0$ for $u > c_0, v > c_0$.

Theorem 4.2. *Let $g \in C^2$ such that $g_{uu} < 0, g_{uv} > 0, g(0,v) \ge 0$, there is $c_0 > 0$ such that $g(u,v) < 0$ for $u > c_0, v > c_0$ and $g_u(0,0) \le \lambda_1$[$h \in C^2$ such that $h_{uv} > 0, h_{vv} < 0, h(u,0) \ge 0$,there is $c_0 > 0$ such that $h(u,v) < 0$ for $u > c_0, v > c_0$ and $h_v(0,0) \le \lambda_1$]. Then there is a number $M(g) > \lambda_1[N(h) > \lambda_1]$ such that for any $h \in S_g$ satisfying $h_v(0,0) > M(g)$ [for any $g \in S_h$ satisfying $g_u(0,0) > N(h)$], (3) has a positive solution in Ω.*

Proof. Suppose $g_u(0,0) \le \lambda_1$. Let $h \in S_g$ be such that $h_v(0,0) > \lambda_1$. Since

$$\begin{aligned}\lim_{c\to\infty} \lambda_1(-g_u(0,\theta_{\frac{c}{-M}}) + g_u(0,0)) &\le \lim_{c\to\infty} \lambda_1(-\inf(g_{uv})\theta_{\frac{c}{-M}} + g_u(0,0)) \\ &\le \lim_{c\to\infty} \lambda_1(-\inf(g_{uv})\frac{c-\lambda_1}{-M}\phi_0 + g_u(0,0)) \\ &= -\infty,\end{aligned}$$

there is a number $M(g) \ge \lambda_1$ such that $\lambda_1(-g_u(0,\theta_{\frac{c}{-M}}) + g_u(0,0)) < g_u(0,0)$ if $c > M(g)$. Hence, if $h_v(0,0) > M(g)$, then $\lambda_1(-g_u(0,\theta_{\frac{h_v(0,0)}{-\inf(h_{vv})}}) + g_u(0,0)) < \lambda_1(-g_u(0,\theta_{\frac{h_v(0,0)}{-M}}) + g_u(0,0)) < g_u(0,0)$.

Let $h_v(0,0) > M(g)$ and $\underline{u} = \epsilon\omega_0$ and $\underline{v} = \theta_{\frac{h_v(0,0)}{-\inf(h_{vv})}}$, where ω_0 is the normalized positive eigenfunction corresponding to $\lambda_1(-g_u(0,\theta_{\frac{h_v(0,0)}{-\inf(h_{vv})}}) + g_u(0,0))$. Then by the Mean Value Theorem, there are $\tilde{u}, \tilde{v}$ such that $0 \le \tilde{u} \le \underline{u}, 0 \le \tilde{v} \le \underline{v}$ and

$$\begin{aligned}g(\underline{u},\underline{v}) - g(0,\underline{v}) &= \underline{u} g_u(\tilde{u},\underline{v}) \\ h(\underline{u},\underline{v}) - h(\underline{u},0) &= \underline{v} h_v(\underline{u},\tilde{v}).\end{aligned}$$

Hence, by the monotonicy of g_u and h_v, for sufficiently small $\epsilon > 0$,

$$\begin{aligned}
& \Delta\underline{u} + g(\underline{u}, \underline{v}) \\
\geq\ & \Delta\underline{u} + g(\underline{u}, \underline{v}) - g(0, \underline{v}) \\
=\ & \Delta\underline{u} + \underline{u}g_u(\tilde{u}, \underline{v}) \\
\geq\ & \Delta\underline{u} + \underline{u}g_u(\underline{u}), \underline{v}) \\
=\ & \Delta\underline{u} + \underline{u}[g_u(0,0) + g_u(\underline{u}, \underline{v}) - g_u(0, \underline{v}) + g_u(0, \underline{v}) - g_u(0,0)) \\
\geq\ & \Delta\underline{u} + \underline{u}[g_u(0,0) + \inf(g_{uu})\underline{u} + g_u(0, \underline{v}) - g_u(0,0)] \\
=\ & \Delta(\epsilon\omega_0) + \epsilon\omega_0[g_u(0,0) + \inf(g_{uu})\epsilon\omega_0 + g_u(0, \theta_{\frac{h_v(0,0)}{-\inf(h_{vv})}}) - g_u(0,0)] \\
=\ & -\epsilon\lambda_1[-g_u(0, \theta_{\frac{h_v(0,0)}{-\inf(h_{vv})}}) + g_u(0,0)]\omega_0 + g_u(0,0)\epsilon\omega_0 + \epsilon^2\omega_0^2 \inf(g_{uu}) \\
=\ & \epsilon\omega_0[g_u(0,0) - \lambda_1(-g_u(0, \theta_{\frac{h_v(0,0)}{-\inf(h_{vv})}}) + g_u(0,0))] + \epsilon^2\omega_0^2 \inf(g_{uu}) \\
>\ & 0 \text{ in } \Omega
\end{aligned}$$

and

$$\begin{aligned}
& \Delta\underline{v} + h(\underline{u}, \underline{v}) \\
\geq\ & \Delta\underline{v} + h(\underline{u}, \underline{v}) - h(\underline{u}, 0) \\
=\ & \Delta\underline{v} + \underline{v}h_v(\underline{u}, \tilde{v}) \\
\geq\ & \Delta\underline{v} + \underline{v}h_v(\underline{u}, \underline{v}) \\
=\ & \Delta\underline{v} + \underline{v}[h_v(0,0) + h_v(\underline{u}, \underline{v}) - h_v(\underline{u}, 0) + h_v(\underline{u}, 0) - h(0,0)] \\
\geq\ & \Delta\underline{v} + \underline{v}(h_v(0,0) + \inf(h_{vv})\underline{v} + h_v(\underline{u}, 0) - h_v(0,0)] \\
=\ & \underline{v}[h_v(\underline{u}, 0) - h_v(0,0)] \\
>\ & 0 \text{ in } \Omega.
\end{aligned}$$

So, $\underline{u}, \underline{v}$ is a lower solution to (3). Hence, by the condition, if $h_v(0,0) > M(g)$, there is a positive solution to (3).

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