4-19-2024

Analyzing the Effect of Targeted Activities on Linear Concept Understanding

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J.N. Andrews Honors Program
Andrews University

HONS 497
Honors Thesis

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4/19/2024

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Abstract

Despite previously showing mastery of all test topics in ALEKS, Andrews' remedial math students continue to struggle with some of them on the paper tests. After previous research, additional teaching activities, including one targeting word problems, were implemented to address student misconceptions about linear equations. My current research compares student performance on the paper tests before and after these activities on questions regarding linear relationship word problems. My findings show no statistically significant difference in student performance after these activities. These results will lead to further curriculum changes, in an attempt to increase students' long-term conceptual understanding and problem-solving skills.

Introduction

Beginning around 2001, there has been a shift in college remedial mathematics courses to self-paced formats that utilize web-based programs. These programs allow for faster feedback and increased personalization of learning, while significantly decreasing student withdrawal rates (Buzzeto-More & Ukoha, 2009). However, research aimed to analyze the differences between performance in a lecture-based class and web-based class has found mixed results on the effectiveness of online programs. Ironsmith et al. (2003) found no significant difference in grades between the two class formats. Additionally, Spradlin and Ackerman (2010) did not find a statistically significant difference in student test scores after adjusting for initial performance on the pretest. On the other hand, Mojarad et al. (2018) showed statistically significant higher pass rates in five different comparisons between students who used ALEKS and those who did not. A case study conducted at Arizona State University found increased pass rates, a decreased achievement gap, and a quicker progression through College Algebra after implementing ALEKS in the class (McGraw Hill Education, n.d.).

While class format has not been shown to affect student performance, other factors have been. Performance depends on student self-efficacy, while many factors—intrinsic goal orientation, task value, self-efficacy, boredom, and frustration—significantly affect course satisfaction (Cho & Heron, 2015). Buzzeto-More and Ukoha (2009) found student satisfaction to be dependent on previous experience with an online course and their attitude towards math in general. Students with a learning goal orientation performed significantly better and were less anxious (Ironsmith et al., 2003).

The remedial courses at Andrews University—MATH091 and MATH092—use the web-based program ALEKS. Students enrolled in these courses are required to attend class, but are given the majority of time in class to work on ALEKS at their own pace. A professor, with teacher's assistants, provides guidance, keeps students on track, and proctors exams. An emporium model of teaching allows for increased personalization of learning (Buzzeto-More & Ukoha, 2009). Additionally, activities have been incorporated into MATH091 and MATH092 to address student attitudes, and thus increase student performance. Currently, remedial mathematics students identify and apply a brain-break, growth mindset phrase, or learning strategy every class period. Brain-breaks may include many different activities, including ones that evoke feelings of calm or focus. Deep breathing exercises, doodling, or hand-eye coordination exercises are some of the most common brain-breaks.

Beginning in the Summer of 2018, Johnston (2021), an honors mathematics and physics student who was a teacher's assistant for MATH091 and MATH092, conducted research analyzing student work on paper exams after mastery of ALEKS topics. Students are required to show evidence of mastery of ALEKS topics, by answering topical questions correctly multiple times, before taking an exam; however, Johnston (2021) identified questions that students were still missing and the errors students were still making with these topics on the exams. Based on her research, changes to the MATH091 and MATH092 curriculum were made in the Fall of 2018 and Spring of 2019. Once a week, students were given a math-based activity designed to combat these common misconceptions. We are now examining the effectiveness of these activities by repeating the analysis process on paper exams taken post-implementation. In the Summer of 2021, Koliadko (2021) analyzed a subset of the problems testing the same mathematical concepts as Johnston, and encouragingly found improvement.

One of the most challenging parts of remedial mathematics is word problems. Daroczy et al. (2015) examined the factors that make word problems difficult for students—including linguistic structure, numerical complexity, and the connections between them. Students often struggle with word problems because of their challenges in translating the natural language of the problem into mathematical language (Ilany & Margolin, 2010). The difficulty in understanding the problem may come from the language, or vocabulary used, and the length of the problem (Sepeng & Sigola, 2013). In their study of plane geometry word problems, Haryanti et al. (2018) found problems understanding the question, choosing a correct mathematical model, choosing correct strategies to solve, and with computational errors. Kingsdorf and Krawec (2014) used the Mayer model to identify several phases of problem solving where errors may occur: translating the problem into a mathematical model or models, integrating models in a logical manner, planning solution strategies, and executing the plan to find a solution.

Many different research studies have proposed ways to decrease student errors in word problem solving. Haryanti et al. (2018) recommend that teachers increase students' exposure to mathematical word problems, so students can learn how to choose a correct mathematical model and develop proper solution strategies. Similarly, Sepeng and Sigola (2013) encourage teachers to explicitly teach problem-solving strategies and provide other visual sources of information for students to draw upon in addition to the written problem. In regards to specifically linear word problems, Stump (2001) proposes that teachers should use a wider range of word problems to illustrate the different representations of slope and listen to students' descriptions of slope to gauge their understanding. While we have found extensive research on student errors in word problem solving, there has not been research done on errors in word problems after students have previously shown mastery in a web-based program.

The goal of this research project is to analyze the effectiveness of the added curriculum activities at reducing student errors on exams, specifically in the area of linear word problems. Exams taken by students in the MATH091 and MATH092 classes before and after the implementation of these activities will be compared to see if there has been a decrease in wrong answers. While Koliadko (2021) found improvement for problems involving linear relationships without a real-life context, this research project will determine if this same success translates to linear word problems.

Methodology

This research project involves examining two sets of exams from students who took MATH091 and MATH092 at Andrews University. It is specifically focused on linear word problems, which represented two problems from the exams – one where students were given a rate (or slope) and the other where students needed to find the rate given the information, and then use the rate to get a final answer. First, the problem is recorded as right, wrong, or skipped based on what the professor had marked. Skipped problems refer to those where the student had been excused from completing the problem because the professor determined they already understood the concept on a previous attempt of the paper exam. Blank answers are considered wrong.

Once the problems have been characterized based on correctness of response, then the wrong answers are analyzed further. To reduce errors in reliability based on bias of the rater, the problems, with the work and step-by-step process, are each completed and then compared to the students' work. The work of the student is then analyzed based on several different possible mistakes or misconceptions. For each mistake, the problem was marked as Yes (present), No (not present), Blank, or Unsure. Thus, a problem could be marked as Yes for several different mistakes if all were present.

After collecting the data, the percentages of wrong answers out of the total answers are compared between the two sets of exams. Standard error and confidence intervals are calculated as well to determine a range of percentage differences. This percentage of difference can be used to determine whether or not students performance improved – these questions were missed less often – after the implemented curriculum activities.

Koliadko (2021) completed the same process with linear non-word problems. Part of this research project will include a comparison of her results (confidence intervals) with those from the linear word problems to determine whether students' improvement varied based on the problem. This will also help in the identification of target areas for improvement.

Results

We calculated various percentages based on student answers from the exams to analyze the effectiveness of the implemented activities on student performance. The implemented activity that specifically targeted student understanding of linear relationships within word problems can be found in Appendix A. Note that the activity involves multiple representations (table, graph, equation) of the linear relationships as well as a contrast between proportional and non-proportional linear relationships. The work on the activity is focused on the relationships rather than the specific exam questions.

Koliadko (2021) analyzed student errors on the non-word problems from exams taken post-activity to compare student performance to the pre-activity exams analyzed by Johnston (2021). Figure 1 shows the overall calculated difference (post-activity minus pre-activity) in percentage of total questions answered wrong and the 95% confidence intervals for three selected non-word problem question types that relate to the linear word problems. (Frequency tables for the intermediary data can be found in B.1.1.1 and B.1.1.2, Appendix B.) All confidence intervals are statistically significant and show an increase in student performance after the implemented activities.

Question Type	Difference	95% CI
Finding Equation of a Line Given Slope and Y-intercept	-8%	(-14%, -2%)
Finding Equation of a Line Given Slope and Point	-10%	(-17%, -3%)
Finding Equation of a Line Given Two Points	-12%	(-21%, -3%)

Figure 1: Linear Non-Word Problems Overall Percentage Incorrect

In comparison, Figure 2 shows my results, which do not indicate statistically significant improvement. (Frequency tables for the intermediary data can be found in B.2.1 and B.2.2, Appendix B.) We cannot conclude from this data that the implemented activity had any effect on student performance on the linear word problems.

Question Type	Difference	95% CI
Given Rate Linear Word Problem	4%	(-4%, 12%)
Not Given Rate Linear Word Problem	0%	(-10%, 11%)

Figure 2: Linear Word Problems Overall Percentage Incorrect

Table B.4.1 in Appendix B shows the detailed analysis of the Given Rate Linear Word Problem by mistake categories. The mistake "incorrect linear equation setup" means that the student correctly identified the problem as requiring a linear relationship but did not place the given information in the correct places. The mistake "arithmetic" refers to any errors students made within solving the model they came up with for a numerical answer. The mistake "treat as direct proportion" means the student interpreted the problem as calling for a proportional relationship, instead of linear. The mistake "decimal representation" refers to any errors made while interpreting time and money as decimals, as well as their unexplained presence or disappearance in a student's work. The mistake "nonlinear attempt" means that the student used some nonlinear form to solve the problem (often multiplying or dividing the numbers present in the problem). Finally, the mistake titled "miscellaneous" refers to any other kind of mistake that was not prevalent enough to warrant its own mistake category. Examples of these mistakes can be found in Section C.1 of Appendix C.

The most frequent mistakes for the Given Rate Linear Word Problem (Tables B.3.1 and B.3.2, Appendix B) were "decimal representation," "incorrect linear equation setup," and "treat as direct proportion." As our activity was not targeting student understanding of decimals within contexts, the results related to the "decimal representation" mistake category do not tell us anything about the effectiveness of the implemented activity in increasing student understanding of linear relationships. Once again, the mistakes "incorrect linear equation setup" and "treat as direct proportion" seem to suggest that students struggle in interpreting and translating the contextual, linguistic situation into the correct mathematical model.

Table B.4.2 in Appendix B shows the detailed analysis of the Not Given Rate Linear Word Problem by mistake categories. The mistake "fixed charge errors" means that the student did not correctly place the fixed charge (or y-intercept) within a linear model (either interpreting it as proportional or placing it within the wrong part of a linear equation). The mistake "treat charge as rate" means the student used the fixed charge (or y-intercept) as the rate (or slope) within their mathematical model of the situation. The mistake "additional cost" refers to errors made with a specifically worded linear word problem that asked students to find the additional cost for

200 guests instead of 150 guests. The mistake "incomplete" means that the student correctly completed a portion of the steps necessary to solve the problem, but gave a premature answer and did not finish solving the problem. All other mistakes have the same explanation as before. Examples of these mistakes can be found in Section C.2 of Appendix C.

The most frequent mistakes for the Not Given Rate Linear Word Problem (Tables B.3.3 and B.3.4, Appendix B) were "treat as direct proportion," "fixed charge errors," and "incorrect linear equation setup." All of these errors have to do with choosing the correct mathematical model and translating from the contextual, linguistic situation to the correctly associated mathematical language.

For comparison, Table B.1.3.1 in Appendix B shows the detailed analysis of the Finding Equation of a Line Given Slope and Y-Intercept question type (linear non-word problem) by mistake categories. (Frequency tables for the intermediary data can be found in B.1.2.1 and B.1.2.2, Appendix B.) Koliadko (2021) and Johnston (2021) provided the following definitions for their mistake categories. The mistake "misrepresenting y-int" means the student found either a correct or incorrect y-intercept, but did not correctly input this into the equation. The mistake "not meaningful linear form" means the equation does not make sense in some way for a linear equation. The mistake "misrepresenting slope" means the student found either a correct or incorrect slope, but then did not correctly input it into the equation. The mistake "finding y-int" means the student did not attempt to find the y-intercept, made errors when trying to calculate the y-intercept, or found a "y-intercept" through an incorrect manner. Finally, the mistake "arithmetic/algebra" means student work had either arithmetic or algebra errors.

The only statistically significant confidence interval for the Finding Equation of a Line Given Slope and Y-Intercept (linear non-word problem) is for the "not meaningful linear form" mistake category. This decrease in errors indicates that students became more familiar with an appropriate linear equation ($y = mx + b$ is the most commonly known one). They were more likely to associate that learned equation with problems asking them to find the equation of a line after the implementation of the activities.

Table B.1.3.2 in Appendix B shows the detailed analysis of the Finding Equation of a Line Given Slope and a Point question type (linear non-word problem) by mistake categories. (Frequency tables for the intermediary data can be found in B.1.2.3 and B.1.2.4, Appendix B.) All these mistakes have the same explanation as before. Again, for this question type, the "not meaningful linear form" mistake is the one with statistically significant improvement.

Table B.1.3.3 in Appendix B shows the detailed analysis of the Finding Equation of a Line Given Two Points question type (linear non-word problem) by mistake categories. (Frequency tables for the intermediary data can be found in B.1.2.5 and B.1.2.6, Appendix B.) Koliadko (2021) and Johnston (2021) provided the following definitions for their mistake categories. The mistake "finding slope" means the students made errors trying to calculate slope or used wrong methods to find a "slope." The mistake "sign issues" means a student made a sign error without any algebra errors. The mistake "slope rule" means the student forgot or messed up the slope formula when calculating slope. The mistake "plugging in point" means the student made some error plugging the point in any area of the calculation (perhaps switched up the x and y variables, etc.). All other mistakes have the same explanation as before.

The statistically significant confidence intervals for the Finding Equation of a

Line Given Two Points (linear non-word problem) are for the "finding y-int," "finding slope," "not meaningful linear form," "slope rule," and "misrepresenting y-int" mistake categories. All of these confidence intervals show a decrease in errors except for the "finding y-int" mistake category, where there was an increase in errors.

Refer to Appendix B for the complete results.

Discussion

The results from analyzing student work on exams pre- and post-activity show that the activity was successful in improving student performance for linear non-word problems. Overall, students missed these questions less often. When considering specific mistakes, one mistake category out of five showed statistically significant improvement for the Finding Equation of a Line Given Slope and Y-Intercept question type, while the other four did not change significantly. The same was true for the Finding Equation of a Line Given Slope and Point question type. For the Finding Equation of a Line Given Two Points question type, four mistake categories out of nine showed statistically significant improvement and one mistake category showed a statistically significant decrease in student performance. This increase in errors could have been due to a shift in focus within the teaching time based on recommendations by Johnston (2021). Her data illustrated a need for "a thorough focus on a flexible understanding of slope" (Johnston, 2021, p. 16). This targeted instruction around slope could mean that students focused on understanding the slope piece, neglecting the y-intercept in the process. Encouragingly, student performance did improve in regards to mistakes related to slope. The statistically significant improvement in finding slope and calculating it through the slope rule suggest increased familiarity with procedures involving calculating slope after the activities. Furthermore, students made less errors with "misrepresenting y-int," meaning more students knew where to plug in their calculated y-intercept into the linear equation. Overall, these results indicate that the implemented activity led to increased familiarity with the correct linear form and where to plug in values.

However, this improvement does not translate to the linear word problems. Overall, there was no significant change in the frequency with which students missed these questions. Even when considering each mistake type individually, none showed statistically significant improvement. The only mistake type that produced a statistically significant result was "decimal representation," and students were making this mistake more frequently, not less. Koliadko's (2021) results suggest the activity led to increased familiarity with a correct linear form, calculating necessary values, and placing values into the linear form. However, this could simply be a result of better memorization on the part of the students or an increased familiarity with the correct form based on repeated exposure to it during activities. This is all procedural knowledge. Students do not need a conceptual understanding of what a linear relationship looks like to remember the formula $y = mx + b$ and know where to plug in given or calculated values.

On the other hand, linear word problems do typically require a conceptual understanding of linear relationships. While students can attempt to memorize a list of steps, the problem-solving nature of word problems require students to identify the appropriate mathematical model for the situation. Students could struggle with this for multiple reasons. They may not be familiar with the situation described in the word problem, leading to troubles visualizing what is being asked of them. Or, English may not be a first language for some students, which could result in a number of difficulties understanding and solving the problem. However, once the student understands the English part of the problem, they must then translate this understanding into mathematical language, choosing a mathematical model. If students have simply memorized that a linear equation is $y = mx + b$, this does not help them decide whether a proportional relationship or a linear relationship is more appropriate for the context. This choice requires a conceptual understanding of what being linear means. My results suggest that these student improvements present in Koliadko's (2021) results did not transfer to word problems. While students may have had a better procedural understanding and knowledge of how to solve non-contextual linear problems, this would not help them if they lacked the conceptual understanding of what it means for something to be a linear relationship. My results indicated that students often struggled with identifying a situation as calling for a linear relationship and understanding what role each given value played within that linear relationship.

When given a rate in a linear word problem, students more easily recognized the situation as a linear relationship. However, the problems where students were not given the rate led to more incorrect answers, despite the questions using words to indicate that there was a unknown constant rate. This could suggest a close connection between students conceptual understanding of slope, or rates, and linear relationships.

In the linear non-word problems, students showed statistically significant improvement on the "not meaningful linear form" mistake type for all question types. The equivalent mistake type for the linear word problems is "nonlinear attempt," but could also could relate to "treat as direct proportion" since the student used an incorrect model. Neither of these mistake types showed statistically significant improvement for the word problems. This once again could suggest that students lack the conceptual understanding of linear relationships. After the implemented activity, students improved in using a meaningful linear form when asked to find an equation of a line. However, the word problems did not explicitly tell students that the relationship requires a linear equation to solve.

Additionally, the linear non-word problems showed statistically significant improvement in mistake categories such as "finding slope," "slope rule," and "misrepresenting y-int." When students attempted the word problems, though, this increased accuracy did not help them improve with mistakes such as "fixed charge errors," "treat charge as rate," and "incorrect linear equation setup." While they correctly calculated a slope using slope rule and plugged in their y-intercept to the linear model during the non-word problems, they did not seem to do any better on the word problems. Students continued to struggle with differentiating between a slope (rate) and a y-intercept (fixed charge) within contexts. This could mean that students have only associated certain procedures with the terms instead of understanding what a slope (or y-intercept) represents and how that changes a linear relationship within contexts.

Based on my research results, I would suggest implementing an increased number of activities over an extended period of time. However, as there are many different topics that students must pass in MATH091/092, this may not always be possible. So, I would also recommend providing an interactive group session either during class, in place of class, or in addition to class for students who need it. I believe, based on my observation of the students over the years as a teacher's assistant, that opportunities for group discussions and problem-solving will lead to a better conceptual understanding of the topic and an increased confidence in their ability to solve the problems. I think that would also be a great opportunity to introduce students to certain problem-solving heuristics and explicitly teach problem-solving processes.

Additionally, I would encourage a discovery learning session with students before they begin working with linear relationships to introduce students to certain concepts (linear, slope, y-intercept, etc.). They could manipulate graphs within a graphing system to see how each piece of the linear model influences the visual representation. Then, students and the teacher could discuss what these changes mean within certain contexts. They could brainstorm real-world linear relationships that they are familiar with and try solving problems within those contexts.

Limitations of this project include some lack of independence due to many of the exams being retakes by the same students - so a student struggling with a question is represented multiple times in the data. There is also the possibility of some differences in analysis of errors by different raters.

Further research is needed on what conceptual knowledge tends to be lacking after successful completion of the standard type of online math mastery questions and interactions that are used.

Acknowledgements

Thank you to the Office of Research and Creative Scholarship for their Undergraduate Research funding over two semesters. Additionally, thank you to Dr. Lynelle Weldon, Lisa Johnston, and Noelle Koliadko for their contributions to this research.

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Appendix A

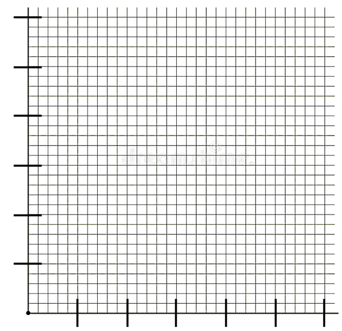
Implemented Activity

M5 NAME:___________________________________

For each scenario, (a) make a table of values, (b) make a graph, (c) give the equation, (d) indicate if it is a direct proportion or not

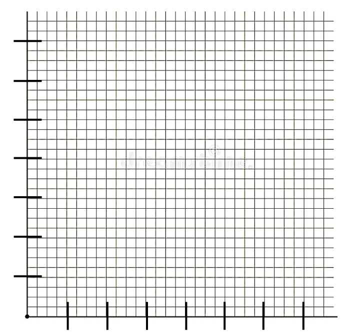
1. Jose earns $10 an hour at S & K Industries. If he works *x* hours in a week, then what is his weekly paycheck?

x	***y***
0	
1	
2	
3	



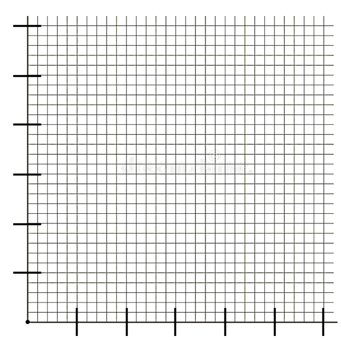
2. At TruGreen, Kelly is on a base pay plus commission and makes $500 per week with 20% commission on the sales made that week. How much does she make with sales of *x* dollars?

x	***y***
0	
100	
500	
1000	



3. The exchange rate is $5 US = 4 Euros. How many Euros can be bought with *x* US dollars?

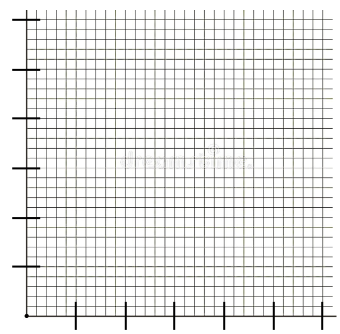
x	***y***
0	
5	
10	
15	



M5 NAME:___________________________________

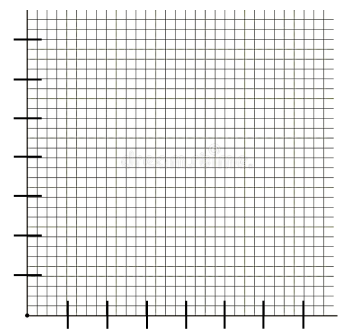
4. At Fresh Market, 10 ounces of cheese costs $2.40. How much does *x* ounces of cheese cost?

x	*y*
0	
5	
10	
15	



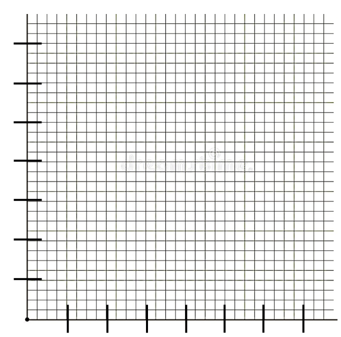
5. Lexie needs to go to church that is *x* miles away. If a taxi costs $2.00 plus $1.50 for each mile, how much will her fare be?

x	*y*
1	
2	
4	
6	



6. Christmas cards cost $1.50 each. How much will she pay for *x* cards?

x	*y*
0	
2	
4	
6	



Appendix B

Complete Results

B.1 - Data From Johnston (2021) and Koliadko (2021)

B.1.1 - Frequency Tables

B.1.1.1 - Pre-Activity

Question Type	Total	Correct	Incorrect	Skipped
Finding Equation of a Line Given Slope and Y-intercept	594	75%	19%	6%
Finding Equation of a Line Given Slope and Point	565	72%	22%	6%
Finding Equation of a Line Given Two Points	599	59%	39%	3%

B.1.1.2 Post-Activity

Question Type	Total	Correct	Incorrect	Skipped
Finding Equation of a Line Given Slope and Y-intercept	119	71%	11%	18%
Finding Equation of a Line Given Slope and Point	119	71%	12%	17%
Finding Equation of a Line Given Two Points	119	62%	27%	11%

B.1.2 - In-depth Analysis

B.1.2.1 - Finding Equation of a Line Given Slope and Y-intercept Pre-Activity

Misconception	# of incorrect answers	% of incorrect answers	% of all responses
misrepresenting y-int	36	32%	6%
not meaningful linear form	34	31%	6%
misrepresenting slope	31	28%	5%
finding y-int	28	25%	5%
arithmetic/algebra	16	14%	3%

B.1.2.2 - Finding Equation of a Line Given Slope and Y-intercept Post-Activity

Misconception	# of incorrect answers	% of incorrect answers	% of all responses
misrepresenting y-int	6	46%	6%
not meaningful linear form	0	0%	0%
misrepresenting slope	3	23%	3%
finding y-int	5	38%	5%
arithmetic/algebra	5	38%	5%

B.1.2.3 - Finding Equation of a Line Given Slope and Point Pre-Activity

Misconception	# of incorrect answers	% of incorrect answers	% of all responses
misrepresenting y-int	35	30%	6%
not meaningful linear form	31	26%	5%
finding y-int	27	23%	5%
misrepresenting slope	24	20%	5%
arithmetic/algebra	14	12%	4%

B.1.2.4 - Finding Equation of a Line Given Slope and Point Post-Activity

Misconception	# of incorrect answers	% of incorrect answers	% of all responses
misrepresenting y-int	6	43%	6%
not meaningful linear form	1	7%	1%
finding y-int	9	64%	9%
misrepresenting slope	3	21%	3%
arithmetic/algebra	3	21%	3%

B.1.2.5 - Finding Equation of a Line Given Two Points Pre-Activity

Misconception	# of incorrect answers	% of incorrect answers	% of all responses
finding y-int	80	35%	13%
arithmetic/algebra	73	32%	12%
finding slope	60	26%	10%
sign issues	50	22%	8%
not meaningful linear form	46	20%	8%
slope rule	24	11%	4%
plugging in point	17	7%	3%
misrepresenting slope	13	6%	2%
misrepresenting y-int	9	4%	2%

B.1.2.6 - Finding Equation of a Line Given Two Points Post-Activity

Misconception	# of incorrect answers	% of incorrect answers	% of all responses
finding y-int	23	72%	22%
arithmetic/algebra	18	56%	17%
finding slope	4	13%	4%
sign issues	4	13%	4%
not meaningful linear form	2	6%	2%
slope rule	1	3%	1%
plugging in point	1	3%	1%
misrepresenting slope	1	3%	1%
misrepresenting y-int	0	0%	0%

B.1.3 - Differences and Confidence Intervals

B.1.3.1 - Finding Equation of a Line Given Slope and Y-intercept

Misconception	Difference	95% CI
misrepresenting y-int	0%	(-5%, 5%)
not meaningful linear form	-6%	(-8%, -4%)
misrepresenting slope	-2%	(-6%, 2%)
finding y-int	0%	(-4%, 4%)
arithmetic/algebra	2%	(-2%, 6%)

B.1.3.2 - Finding Equation of a Line Given Slope and Point

Misconception	Difference	95% CI
misrepresenting y-int	0%	(-5%, 5%)
not meaningful linear form	-4%	(-7%, -1%)
finding y-int	4%	(-1%, 9%)
misrepresenting slope	-2%	(-6%, 2%)
arithmetic/algebra	-1%	(-4%, 2%)

B.1.3.3 - Finding Equation of a Line Given Two Points

Misconception	Difference	95% CI
finding y-int	9%	(1%, 17%)
arithmetic/algebra	5%	(-2%, 12%)
finding slope	-6%	(-10%, -2%)
sign issues	-4%	(-8%, 0%)
not meaningful linear form	-6%	(-9%, -3%)
slope rule	-3%	(-5%, -1%)
plugging in point	-2%	(-4%, 0%)
misrepresenting slope	-1%	(-3%, 1%)
misrepresenting y-int	-2%	(-3%, -1%)

B.2 - Frequency Tables

B.2.1 - Pre-Activity

Question Type	Total	Correct	Incorrect	Skipped
Given Rate Linear Word Problem	491	75%	21%	5%
Not Given Rate Linear Word Problem	363	42%	55%	3%

B.2.2 - Post-Activity

Question Type	Total	Correct	Incorrect	Skipped
Given Rate Linear Word Problem	127	66%	24%	9%
Not Given Rate Linear Word Problem	111	38%	55%	7%

B.3 - In-depth Analysis

B.3.1 - Given Rate Linear Word Problem Pre-Activity

Misconception	# of incorrect answers	% of incorrect answers	% of all responses
incorrect linear equation setup	28	28%	6%
arithmetic	10	10%	2%
treat as direct proportion	8	8%	2%
decimal representation	31	31%	7%
nonlinear attempt	4	4%	1%
miscellaneous	1	1%	0%

B.3.2 - Given Rate Linear Word Problem Post-Activity

Misconception	# of incorrect answers	% of incorrect answers	% of all responses
incorrect linear equation setup	12	39%	10%
arithmetic	2	6%	2%
treat as direct proportion	5	16%	4%
decimal representation	16	52%	14%
nonlinear attempt	1	3%	1%
miscellaneous	1	3%	1%

B.3.3 - Not Given Rate Linear Word Problem Pre-Activity

Misconception	# of incorrect answers	% of incorrect answers	% of all responses
fixed charge errors	71	36%	20%
treat as direct proportion	89	45%	25%
treat charge as rate	3	2%	1%
incorrect linear equation setup	68	34%	19%
arithmetic	8	4%	2%
additional cost	28	14%	8%
incomplete	10	5%	3%
miscellaneous	3	2%	1%

B.3.4 - Not Given Rate Linear Word Problem Post-Activity

Misconception	# of incorrect answers	% of incorrect answers	% of all responses
fixed charge errors	25	41%	24%
treat as direct proportion	31	51%	30%
treat charge as rate	2	3%	2%
incorrect linear equation setup	17	28%	17%
arithmetic	2	3%	2%
additional cost	14	23%	14%
incomplete	4	7%	4%
miscellaneous	2	3%	2%

B.4 - Differences and Confidence Intervals

B.4.1 - Given Rate Linear Word Problem

Misconception	Difference	95% CI
incorrect linear equation setup	4%	(-1%, 10%)
arithmetic	0%	(-3%, 2%)
treat as direct proportion	3%	(-1%, 6%)
decimal representation	7%	(1%, 14%)
nonlinear attempt	0%	(-2%, 2%)
miscellaneous	1%	(-1%, 2%)

B.4.2 - Not Given Rate Linear Word Problem

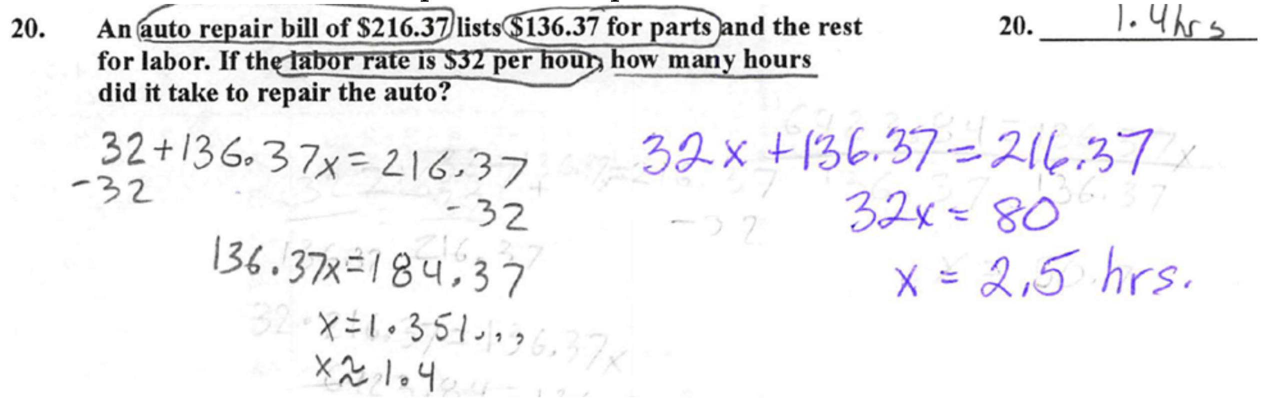
Misconception	Difference	95% CI
fixed charge errors	4%	(-5%, 13%)
treat as direct proportion	5%	(-5%, 14%)
treat charge as rate	1%	(-2%, 4%)
incorrect linear equation setup	-3%	(-11%, 5%)
arithmetic	0%	(-3%, 3%)
additional cost	6%	(-1%, 13%)
incomplete	1%	(-3%, 5%)
miscellaneous	1%	(-2%, 4%)

Appendix C

Examples of Student Mistakes

C.1 - Given Rate

C.1.1 - Incorrect Linear Equation Setup



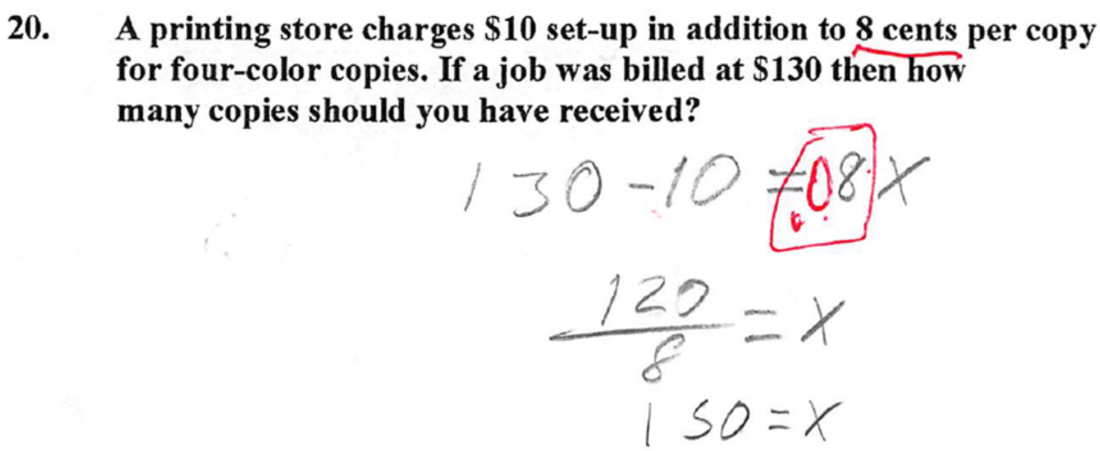
20. **An auto repair bill of $216.37 lists $136.37 for parts and the rest for labor. If the labor rate is $32 per hour, how many hours did it take to repair the auto?**

20. 1.4hrs

$32 + 136.37x = 216.37$
$-32 \qquad\qquad -32$
$136.37x = 184.37$
$x = 1.351...$
$x \approx 1.4$

$32x + 136.37 = 216.37$
$32x = 80$
$x = 2.5$ hrs.

C.1.2 - Arithmetic



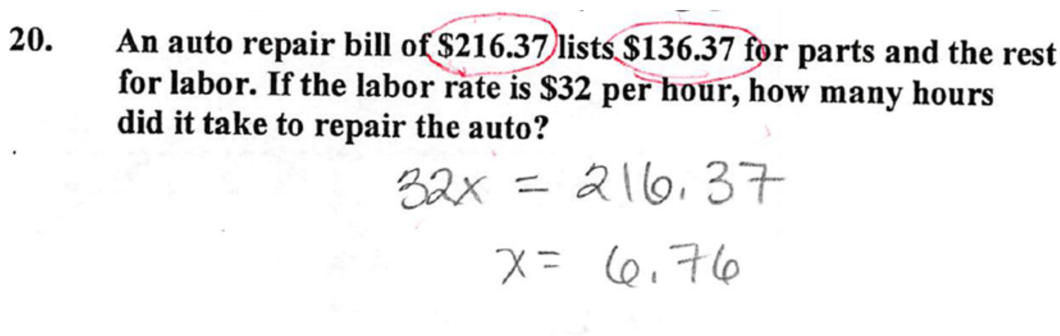
20. **A printing store charges $10 set-up in addition to 8 cents per copy for four-color copies. If a job was billed at $130 then how many copies should you have received?**

$130 - 10 = .08x$

$\frac{120}{8} = x$

$150 = x$

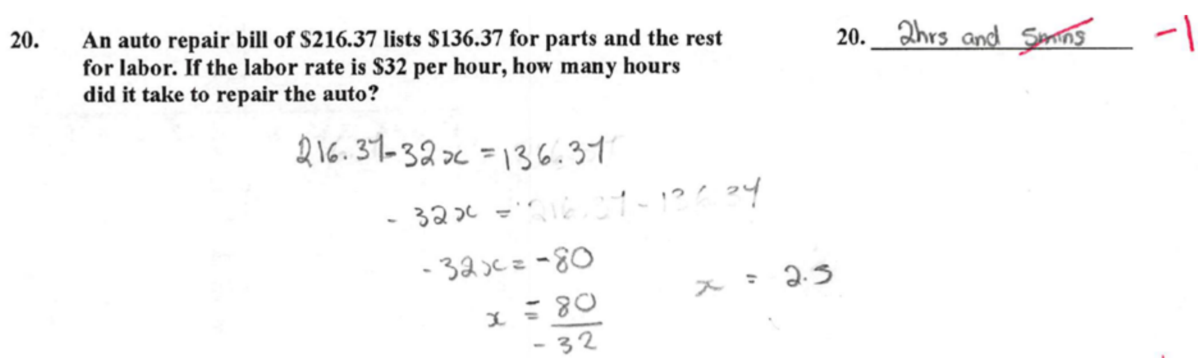
C.1.3 - Treat as Direct Proportion


20. An auto repair bill of $216.37 lists $136.37 for parts and the rest for labor. If the labor rate is $32 per hour, how many hours did it take to repair the auto?

$32x = 216.37$

$x = 6.76$

C.1.4 - Decimal Representation


20. An auto repair bill of $216.37 lists $136.37 for parts and the rest for labor. If the labor rate is $32 per hour, how many hours did it take to repair the auto?

20. 2hrs and 5mins −1

$216.37 - 32x = 136.37$

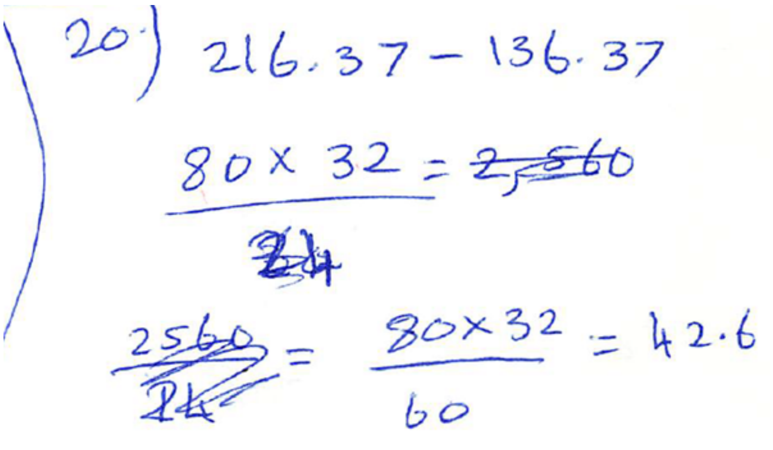
$-32x = 216.37 - 136.37$

$-32x = -80$

$x = \frac{80}{-32}$

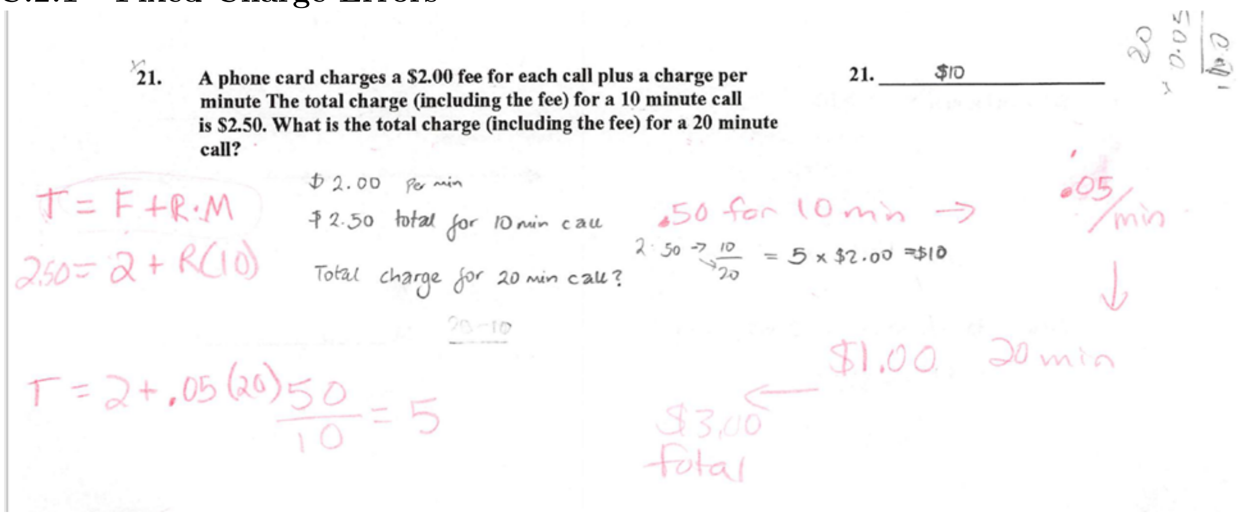
$x = 2.5$

C.1.5 - Nonlinear Attempt & Miscellaneous



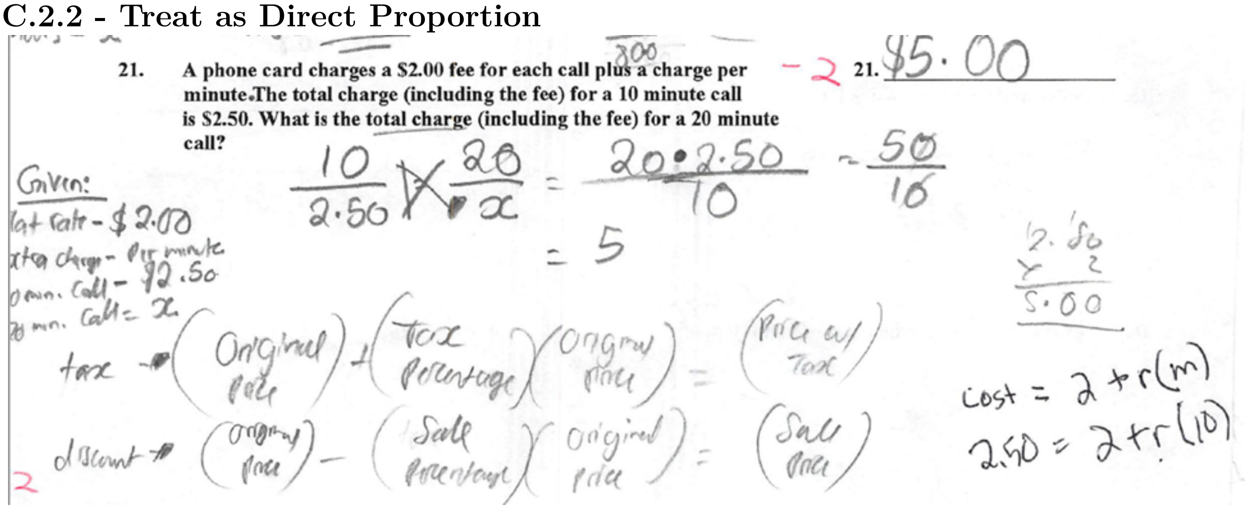
C.2 - Not Given Rate

C.2.1 - Fixed Charge Errors


21. A phone card charges a $2.00 fee for each call plus a charge per minute The total charge (including the fee) for a 10 minute call is $2.50. What is the total charge (including the fee) for a 20 minute call?

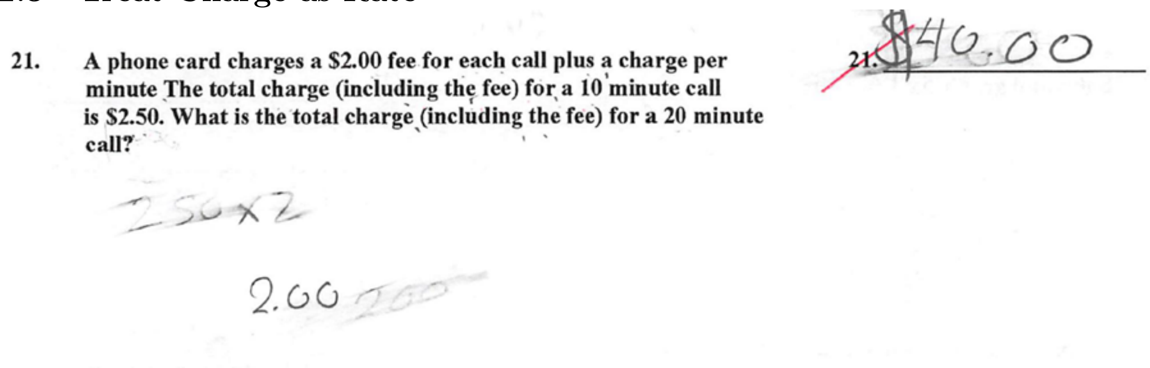
21. $10

C.2.2 - Treat as Direct Proportion


21. A phone card charges a $2.00 fee for each call plus a charge per minute. The total charge (including the fee) for a 10 minute call is $2.50. What is the total charge (including the fee) for a 20 minute call?

21. $5.00

C.2.3 - Treat Charge as Rate

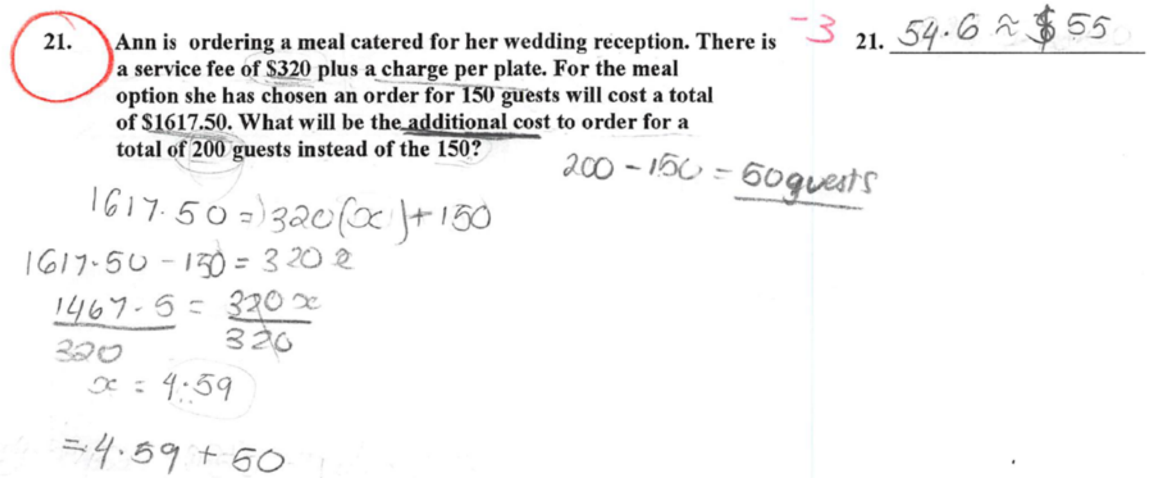

21. A phone card charges a $2.00 fee for each call plus a charge per minute The total charge (including the fee) for a 10 minute call is $2.50. What is the total charge (including the fee) for a 20 minute call?

21. $40.00

2.50 x 2

2.00

C.2.4 - Incorrect Linear Equation Setup


21. Ann is ordering a meal catered for her wedding reception. There is a service fee of $320 plus a charge per plate. For the meal option she has chosen an order for 150 guests will cost a total of $1617.50. What will be the additional cost to order for a total of 200 guests instead of the 150?

-3 21. 54.6 ≈ $55

200 - 150 = 50 guests

1617.50 = 320(x) + 150

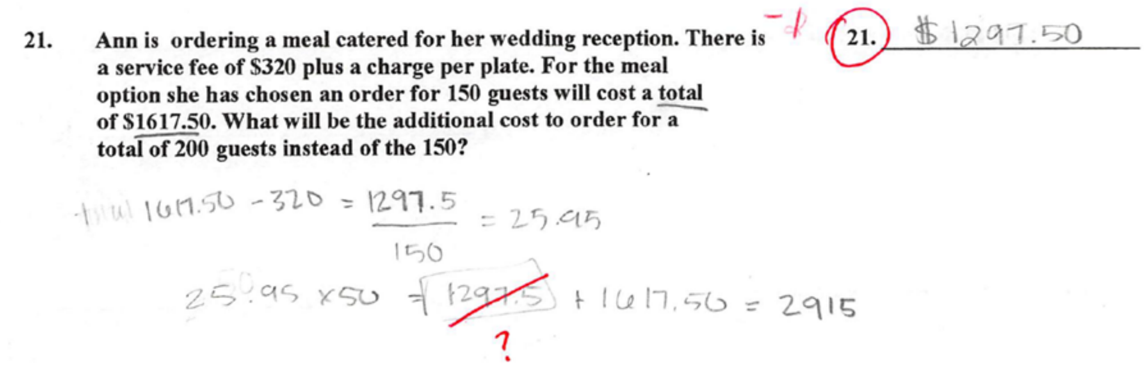
1617.50 - 150 = 320x

1467.5 / 320 = 320x / 320

x = 4.59

= 4.59 + 50

C.2.5 - Arithmetic



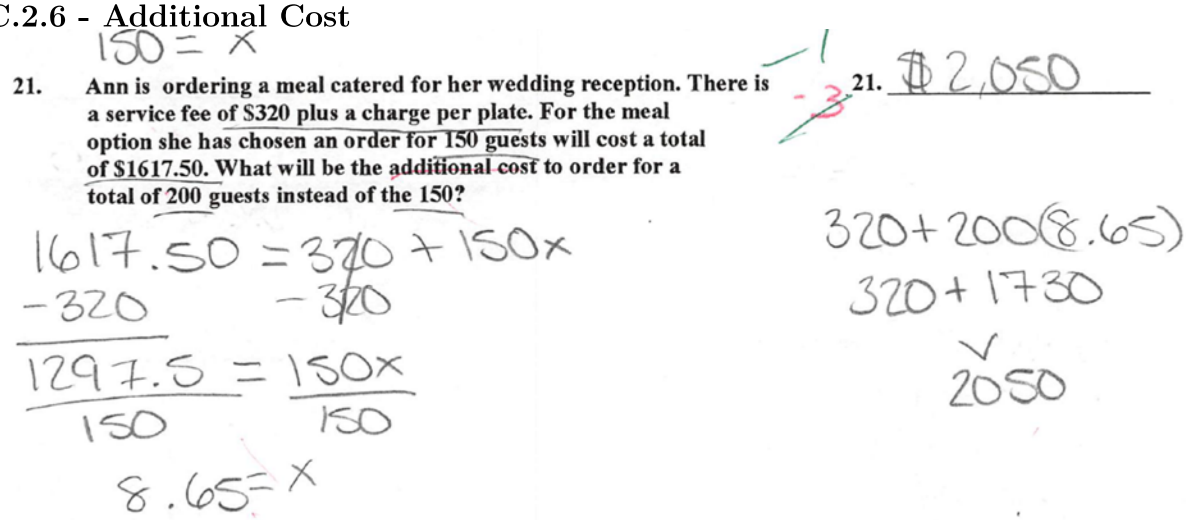
21. Ann is ordering a meal catered for her wedding reception. There is a service fee of $320 plus a charge per plate. For the meal option she has chosen an order for 150 guests will cost a total of $1617.50. What will be the additional cost to order for a total of 200 guests instead of the 150?

21. $1297.50

1617.50 − 320 = 1297.5 / 150 = 25.95

25.95 × 50 = 1297.5 + 1617.50 = 2915

C.2.6 - Additional Cost



150 = x

21. Ann is ordering a meal catered for her wedding reception. There is a service fee of $320 plus a charge per plate. For the meal option she has chosen an order for 150 guests will cost a total of $1617.50. What will be the additional cost to order for a total of 200 guests instead of the 150?

21. $2,050

1617.50 = 320 + 150x

−320 −320

1297.5 / 150 = 150x / 150

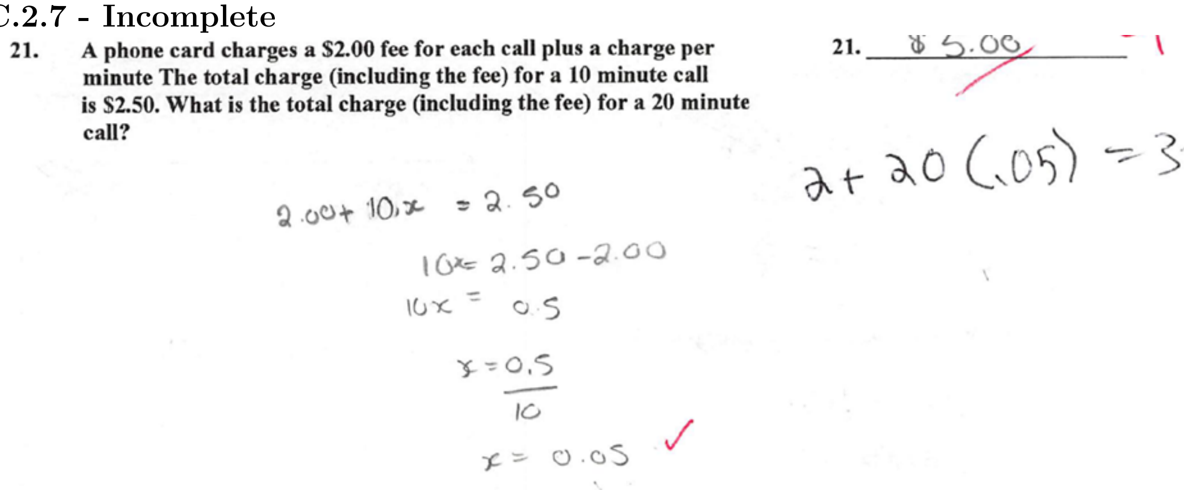
8.65 = x

320 + 200(8.65)

320 + 1730

2050

C.2.7 - Incomplete



21. A phone card charges a $2.00 fee for each call plus a charge per minute The total charge (including the fee) for a 10 minute call is $2.50. What is the total charge (including the fee) for a 20 minute call?

21. $5.00 −1

2 + 20 (.05) = 3

2.00 + 10x = 2.50

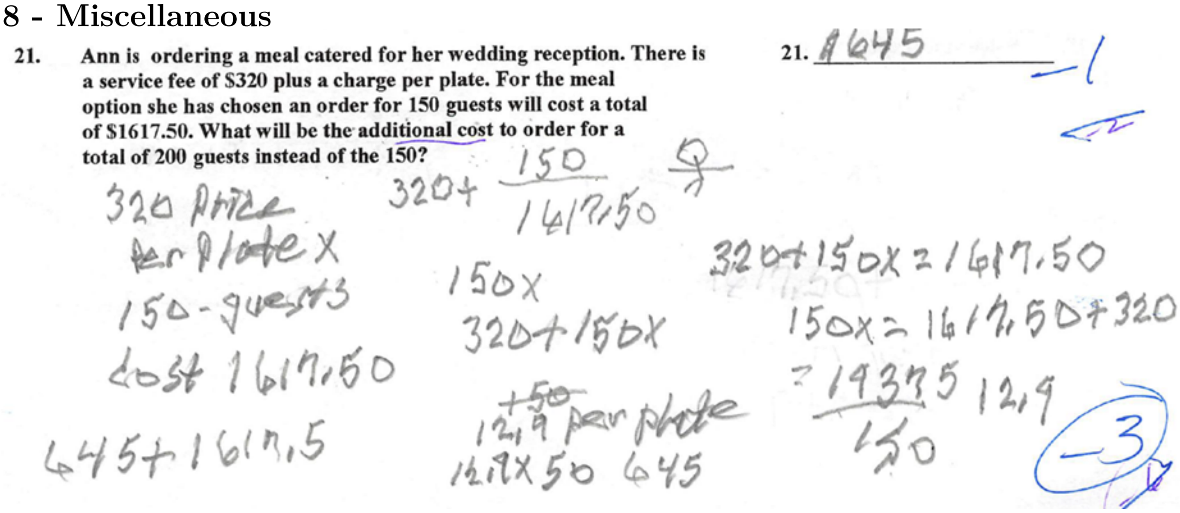
10x = 2.50 − 2.00

10x = 0.5

x = 0.5 / 10

x = 0.05 ✓

C.2.8 - Miscellaneous



21. Ann is ordering a meal catered for her wedding reception. There is a service fee of $320 plus a charge per plate. For the meal option she has chosen an order for 150 guests will cost a total of $1617.50. What will be the additional cost to order for a total of 200 guests instead of the 150?

21. $645 −1

320 price

per plate x

150 - guests

cost 1617.50

645 + 1617.5

320 + 150/1617.50

150x

320 + 150x

+50

12.9 per plate

12.9 x 50 645

320 + 150x = 1617.50

150x = 1617.50 + 320

= 1937.5 / 150 12.9

−3