Bifurcation Analysis of a Discrete-time Model for Seabird Reproduction

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Abstract

Glaucous-winged gulls (*Larus glaucescens*) breed in a large colony on Protection Island, Washington, and are known to exhibit every-other-day egg-laying synchrony in dense areas of the colony. We present a discrete-time model for egg-laying and use Jury Conditions to find the stability of the system as a function of the crowding factor. The equilibrium loses stability when the crowding factor exceeds a critical value, and the system begins synchronous stable oscillations. We also explore the effects of synchrony in the presence of egg predation and show that synchrony can be advantageous for the population.

Methodology

Steps in the deductive method:

1. Study the problem (play with it, probe it, experiment with ideas)
2. Find patterns
3. Develop a hypothesis (called a conjecture)
4. Prove conjecture as a theorem

A mathematical proof is a finite set of deductive steps following from axioms, definitions, and previously proved theorems that are used to arrive at the statement being proved. Mathematical research consists of the theorems and proofs constructed by the researcher.

Model

- The model is discrete-time with time step of 1 day
- System of equations represents biological system
- Three distinct classes of female gulls
    1. w: birds not yet ovulating
    2. x: birds in first day of ovulation cycle
    3. y: birds in second day of ovulation cycle
- At each time step b birds enter system
- Gulls begin ovulation with probability dependent on nesting density c
- Each bird moves from x class to y class with probability 1
- Either repeats ovulation cycle with probability p or leaves the system with probability 1 - p to begin incubation

Model Continued

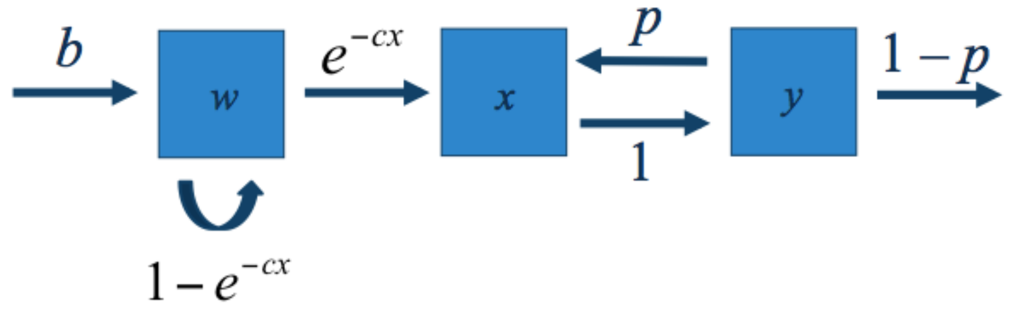

Figure 1: Discrete-time model for biological system representing three distinct classes of birds

$$w_{t+1} = b + \left(1 - e^{-cx_t}\right) w_t$$

$$x_{t+1} = py_t + w_t e^{-cx_t}$$

$$y_{t+1} = x_t$$

Equation 1: Model equations for biological system

- Two simplifying mathematical assumptions
    1. number of birds entering the system has no limit
    2. breeding season is infinitely long
- I used the methods of dynamical systems theory to:
    1. identify the steady states of the model
    2. find the conditions under which steady states are stable
    3. determine whether egg-laying synchrony is beneficial at population level

Results

- Two steady states: equilibrium and two-cycle
    - Equilibrium: solution that does not change with time
    - Two-cycle: state in which the system oscillates between two fixed values
- Apply Jury Conditions to system of equations to find critical value c_1 at which equilibrium loses stability
- At c_1 system begins to oscillate between two values, corresponding to egg-laying synchrony

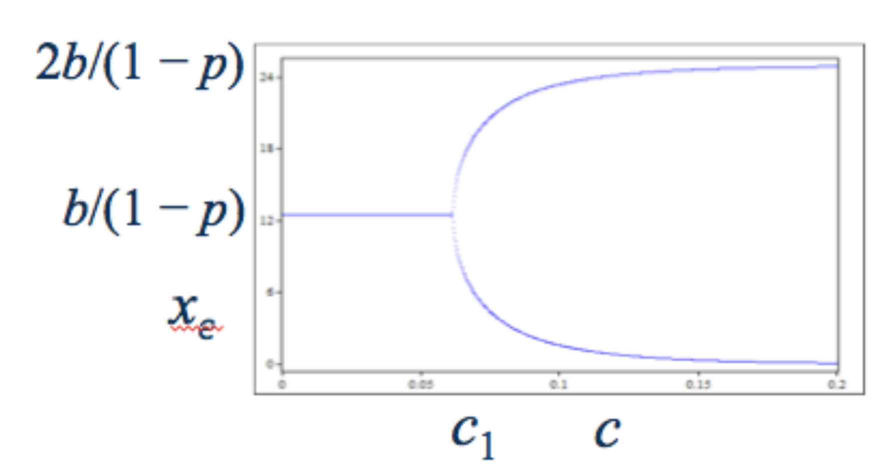

Figure 2: Bifurcation diagram showing steady states of system as a function of crowding factor c

Conclusions

In my research project I was able to show using the deductive method of mathematical research that there are two steady states for the biological system. Egg-laying synchrony occurs when the colony density exceeds the critical value as shown in the bifurcation diagram. Also, I was able to prove that, for this model, egg-laying synchrony is beneficial to the population. Intuitively, this is because the proportion of eggs cannibalized decreases due to predator satiation when many eggs are laid synchronously.

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