A Periodic Matrix Model of Seabird Behavior and Population Dynamics

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Abstract

Rising sea surface temperatures (SSTs) in the Pacific Northwest lead to food resource reductions for surface-feeding seabirds, and have been correlated with several marked behavioral changes. Namely, higher SSTs are associated with increased egg cannibalism and egg-laying synchrony in the colony. We study the long-term effects of climate change on population dynamics and survival by considering a simplified, cross-season model that incorporates both of these behaviors in addition to density-dependent and environmental effects. We show that cannibalism can lead to backward bifurcations and strong Allee effects, allowing the population to survive at lower resource levels than would be possible otherwise.

Background

- Increases in SSTs in the Pacific Northwest depress marine food webs, leading to decreased resource levels for surface-feeding seabirds such as glaucous-winged gulls (*Larus glaucescens*)
- Decreased resource levels lead to:
  - increased egg cannibalism
  - increased egg-laying synchrony
- What are the consequences of these changes on long-term population persistence and dynamics?
- The presence of backward bifurcations would imply that the population can survive at a low resource level, in a region for which the net reproductive number $R_0 < 1$
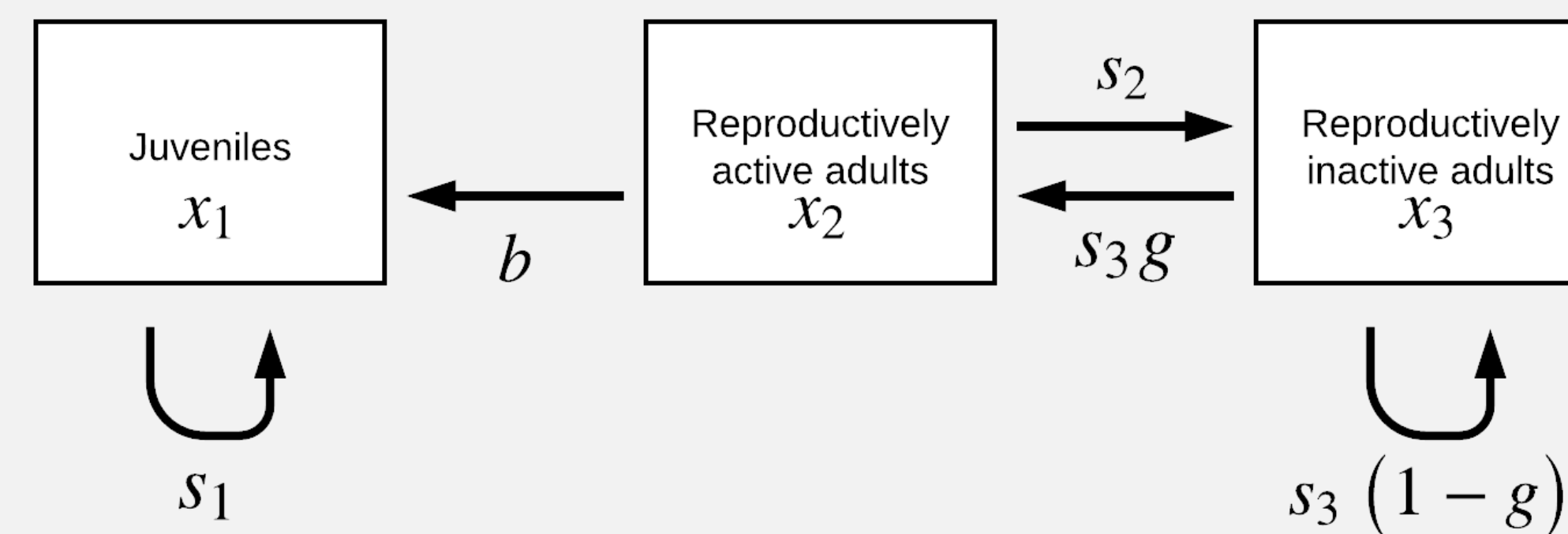
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Model and Analysis

Figure 1: Within-season dynamics

The model is a k−periodically−forced, nonlinear, three-dimensional map

$$\mathbf{x}(t+1) = C\big(t, \mathbf{x}(t)\big)\mathbf{x}(t)$$

where the time step is one day and

$$\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} \text{juveniles} \\ \text{reproductively active adults} \\ \text{reproductively inactive adults} \end{pmatrix}.$$

The projection matrix

$$C(t,\mathbf{x}) = \begin{cases} W(\mathbf{x}) & \text{for } t = 0,1,\dots,k-2 \\ A & \text{for } t = k-1 \end{cases}$$

is extended periodically for all $t \geq 0$. The within-season and between-season projection matrices are, respectively,

$$W(\mathbf{x}) = \begin{pmatrix} s_1 f(1-\pi_2(x_2)x_2)(1-\pi_3(x_3)x_3) & bf\varphi(\mathbf{x}) & 0 \\ 0 & 0 & s_3 f \beta_3(\mathbf{x})g(x_2) \\ 0 & s_2 f \beta_2(\mathbf{x}) & s_3 f \beta_3(\mathbf{x})\big(1-g(x_2)\big) \end{pmatrix}$$

$$A = \begin{pmatrix} 0 & 0 & 0 \\ \nu & \nu & \nu \\ 1-\nu & 1-\nu & 1-\nu \end{pmatrix}.$$

- We implemented the model in MATLAB in order to create bifurcation diagrams with ρ as the bifurcation parameter
- We also studied it analytically:
  - the model can be reduced to an autonomous scalar map of the form $P_{n+1} = r(P_n)P_n$, where $P = x_1 + x_2 + x_3$
  - we found $r(P)$ for $k = 3$, and linearized it around an equilibrium P_e
  - the primary bifurcation point occurs when $r(0) = 1$
    - $\dfrac{\mathrm{d}r}{\mathrm{d}P}(0) > 0 \Rightarrow \lambda > 1$ and the positive branch is unstable
    - $\dfrac{\mathrm{d}r}{\mathrm{d}P}(0) < 0 \Rightarrow \lambda < 1$ and the positive branch is stable
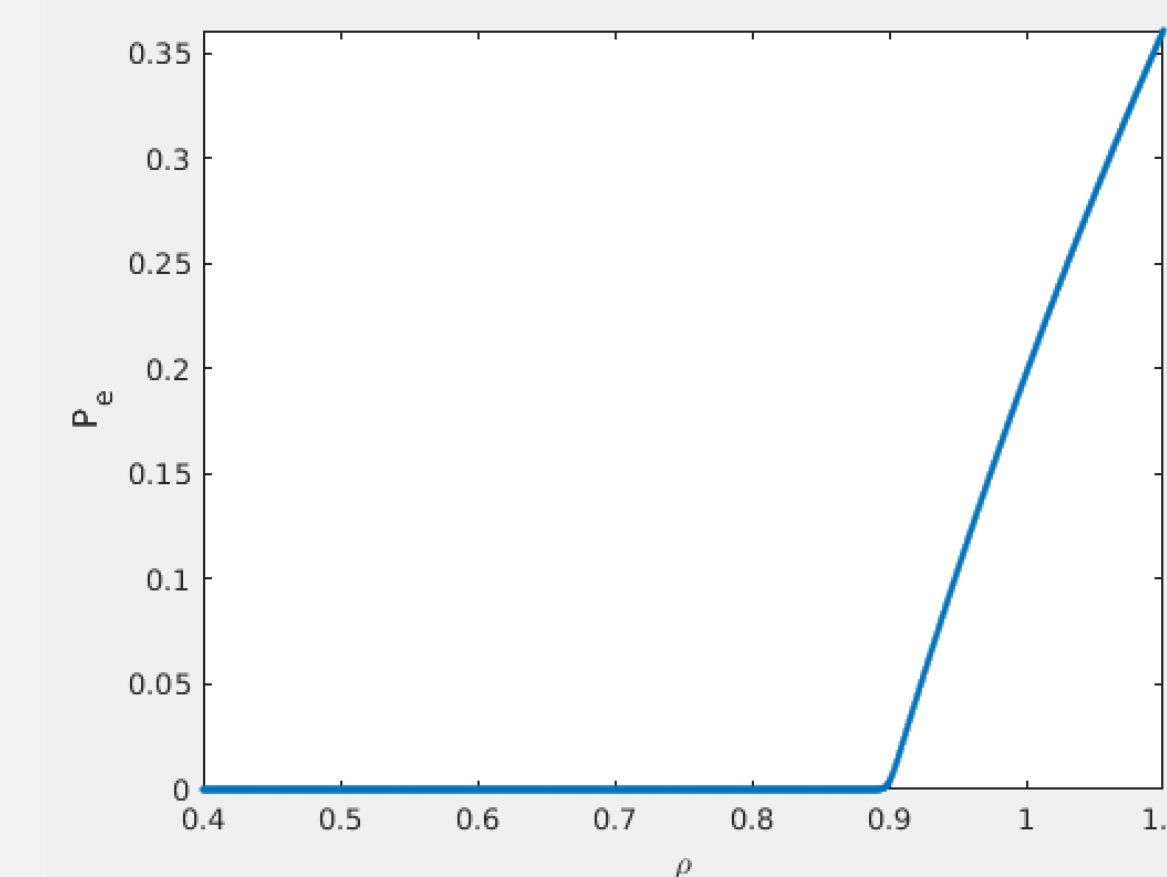
Conclusions

Figure 2: Without cannibalism, stable (forward) bifurcation
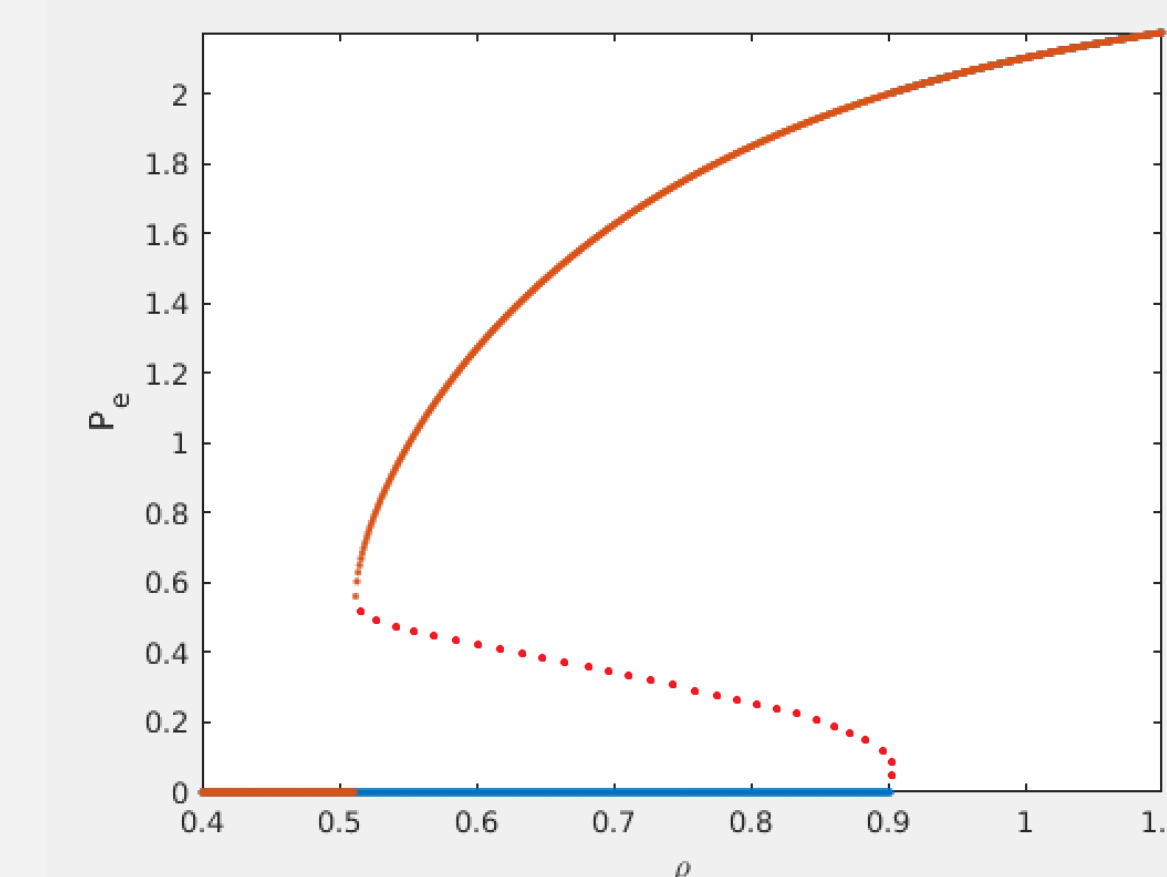

Figure 3: With cannibalism, unstable (backward) bifurcation

Theorems ($k = 3$):
- The net effect of synchrony is negative.
- Greater ρ values are needed for the onset of a stable equilibrium in the presence of full synchrony than in all other cases.
- Backward bifurcations can arise from cannibalism alone, but not from synchrony alone.

Selected Bibliography

Hayward, J. L. *et al.* Egg cannibalism in a gull colony increases with sea surface temperature. *The Condor* **116**, 62–73 (2014).

Henson, S. M., Hayward, J. L., Cushing, J. M. & Galusha, J. C. Socially induced synchronization of every-other-day egg laying in a seabird colony. *The Auk* **127**, 571–580 (2010).

Henson, S. M., Cushing, J. M. & Hayward, J. L. Socially induced ovulation synchrony and its effect on seabird population dynamics. *Journal of Biological Dynamics* **5**, 495–516 (2011).

Weir, S. *Ovulation Synchrony as an Adaptive Response to Egg Cannibalism in a Seabird Colony.* Honors thesis, Andrews University (2015).